A Mathematical Model of Tooth Geometry and Undercutting Condition 
and Contact Ratio Analysis of the Cosine Gear Drive Generated by a Cosine Rack Cutter

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Abstract

This paper proposes a cosine gear drive provided with a pair of cosine gears which are generated by a cosine rack cutter. Based on the coordinate transformation theory and the theory of gearing, a mathematical model of tooth geometry of the cosine gear drive is developed. The condition of undercutting for the cosine gear generated by a cosine rack cutter is established to determine the minimum number of teeth of non-undercutting. An analytic mathematical model for evaluating contact ratio of the cosine gear drive is built. A cause-and-effect analysis for contact ratio is executed under considering gear module, gear ratio, addendum coefficient, and dedendum coefficient as causes. Results of the analysis show that gear module does not affect contact ratio at all but the other causes do affect contact ratio. The proposed cosine gear drive is able to reduce the size of gear box since the minimum number of teeth of non-undercutting is only 6. Furthermore, the production cost and production lead time of the proposed cosine gear drive are low because it can be manufactured efficiently and economically by regular hobbing machines. The use of CNC machines is not necessary.

Keywords: Cosine Gear Drive, Contact Ratio, Rack Cutter, Tooth Geometry, Undercutting

1. Introduction

It goes without saying that involute gears are the most important and popular type of gears in modern industry. Involute gears can be manufactured easily and efficiently by hob cutters on hobbing machines. Therefore, the production cost of involute gears is low. Moreover involute gears are not sensitive to center distance variations. Although involute gears have many advantages, a serious limitation of involute gears is that the minimum number of teeth of non-undercutting cannot be too small. For example, if a rack cutter provided with a standard pressure angle of 20 degree and a standard tooth height is applied to generate an involute gear without any modification, the minimum number of teeth of non-undercutting is 17. Therefore, when facing a mission to reduce the size of gear box as much as possible, designers may consider to replace the traditional involute gear drive by the cosine gear drive proposed in this paper since the minimum number of teeth of non-undercutting of the proposed cosine gear is much smaller than that of involute gear.

This paper, inspired by the manufacturing way of involute gears, proposes a new version of cosine gear drive which is generated by a cosine rack cutter. The proposed cosine gear drive can also be manufactured efficiently by using hobbing method. Therefore, the cost of production and the lead time of production are low. Although Luo et al. [1] have already proposed another version of cosine gear drive, their cosine gear drive is not generated by a rack cutter. The application of CNC machines is required to manufacture their cosine gear drive. Yu et al. [2] conducted a contact ratio analysis for the version of cosine gear drive proposed by Luo et al. Wang et al. [3] studied the effect of assembly errors on contact stress and bending stress for the cosine gear drive proposed by Luo et al. Tsai and Tsai [4] applied quadratic parametric profile to design high contact ratio spur gears. Fong et al. [5] proposed using the line of action to inversely determine the parametric tooth profile of spur gear. Tsay and Fong [6] studied a helical gear drive which is composed of an involute gear and a circular-arc pinion. Komori et al. [7] created a logix tooth profile which has zero relative curvature at many contact points. Zhang et al. [8] proposed a double involute gear, the profile of which is composed of two involute curves. Ariga and Nagata [9] proposed a new Wildhaber-Novikov gear drive which is not sensitive to center distance errors by applying a basic rack cutter provided with circular-arc and involute profiles. Lee [10] proposed a manufacturing process for a cylindrical crowned gear drive to obtain a controllable fourth order polynomial function of transmission errors. Lee et al. [11] proposed a systematic method.
to control intelligently and precisely the magnitude of parabolic-like transmission error of a pair of gears by solving a system of nonlinear equations. Zhang and Wang [12] used tooth contact analysis method to study the influences of installation errors on double circular arc tooth spiral bevel gear.

This paper is organized as follows. Firstly, a mathematical model of tooth geometry of the cosine gear drive which is generated by a cosine rack cutter is established based on the coordinate transformation theory and the theory of gearing. Secondly, the condition of undercutting for the proposed cosine gear is developed and the minimum number of teeth of non-undercutting is solved. Thirdly, an analytic mathematical model for the analysis of contact ratio is proposed. Fourthly, a numerical example of the proposed cosine gear drive is presented and a cause-and-effect analysis for contact ratio is executed. Finally, discussions on how the causes affect contact ratio are made.

2. Mathematical Model of Tooth Geometry of Cosine Gears

According to the coordinate transformation theory and the theory of gearing [13], the mathematical model of tooth geometry of cosine gears can be obtained by using the tooth geometry of generating tool and the relative motion between the generating tool and the generated gears. Shown in Figure 1 is the cosine rack cutter used to be the generating tool of cosine gears. Since the tooth profile of the rack cutter is a cosine function, the rack cutter is given the name of cosine rack cutter. To represent the mathematical model of tooth geometry of the cosine rack cutter, a coordinate system \( S_{xyz} \) is applied to connect rigidly to the rack cutter. The tooth geometry of the cosine rack cutter can be represented in \( S_{xyz} \) by the following equations:

\[
r_r(u, v) = u \mathbf{i} - h_f \cos(2u/m) \mathbf{j} + v \mathbf{k},
\]

(1)

Normal vector of the tooth geometry is determined by

\[
N_r(u) = \left( \frac{\partial r_r}{\partial u} \times \frac{\partial r_r}{\partial v} \right) = 2(h_f/m) \sin(2u/m) \mathbf{i} - \mathbf{j} + 0 \mathbf{k},
\]

(2)

Here, \( h_f \) denotes dedendum tooth depth and \( m \) denotes gear module. The relative motion between the generating rack cutter and the generated cosine gears are shown in Figure 2. Two coordinate system \( S_{xyz} \) and \( S'_{xyz} \) are applied to connect rigidly to the lower cosine gear \( G_1 \) and the upper cosine gear \( G_2 \), respectively. In the process of generation, the cosine rack cutter translates horizontally to the left, the lower cosine gear \( G_1 \) rotates counterclockwise, and the upper cosine gear \( G_2 \) rotates clockwise. Centrod of the rack cutter and centroids of the gears roll without sliding with each other. Thus, the following condition is observed:

\[
s = R_1 \phi_1 = R_2 \phi_2
\]

(3)

where \( s \) is translational parameter of the cosine rack cutter; \( R_1 \) is radius of centroid of gear \( G_1 \); \( R_2 \) is radius of centroid of gear \( G_2 \); \( \phi_1 \) is rotational parameter of gear \( G_1 \); \( \phi_2 \) is rotational parameter of gear \( G_2 \). The tooth geometry of rack cutter will form a family of surfaces in coordinate system \( S_{xyz} \) and the family of surfaces is represented in \( S_{xyz} \) by

\[
r_i = \begin{bmatrix}
x_i \\
y_i \\
z_i 
\end{bmatrix} = \begin{bmatrix}
(u - R_i \phi_i) \cos \phi_i + [R_i - h_f \cos(2u/m)] \sin \phi_i \\
R_i - h_f \cos(2u/m) \cos \phi_i + (R_i \phi_i - u) \sin \phi_i \\
\end{bmatrix}
\]

(4)

The relative velocity between the generating rack cutter and the generated gear \( G_1 \) is represented by

\[
V_r^{(i)}(u, \phi_i) = V_{x_r}^{(i)} \mathbf{i} + V_{y_r}^{(i)} \mathbf{j} + V_{z_r}^{(i)} \mathbf{k} = -\omega_1 h_f \cos(2u/m) \mathbf{k} + \omega_1 (R_i \phi_i - u) \mathbf{j} + 0 \mathbf{k},
\]

(5)

where \( \omega_1 \) is angular velocity of gear \( G_1 \). Based on the theory of gearing [13], the necessary condition for the existence of envelope to the family of surfaces represented by Eq. (4) is the following equation of meshing:

\[
\Phi_i(u, \phi_i) = N_r(u) \cdot V_r^{(i)}(u, \phi_i) = \omega_1 \left( u - R_i \phi_i - \frac{h_f^2 \sin(4u/m)}{m} \right) = 0
\]

(6)

Since \( \omega_1 \) is not equal to zero, the parameter \( \phi_i \) in Eq. (6) can be solved out and can be represented by a function of \( u \) as follows:
The mathematical model of tooth geometry of cosine gear $G_1$ can then be obtained as follows:

$$\phi_1 = \frac{1}{R_1} \left( u - \frac{1}{m} h_j^2 \sin(4u/m) \right)$$  \hspace{1cm} (7)

Similarly, the surface of the cosine rack cutter will also form a family of surfaces in $S_3(x_2, y_2, z_2)$ and the family of surfaces is represented in $S_3(x_2, y_2, z_2)$ by

$$r_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} (u - R_2 \phi_2) \cos \phi_2 + [R_2 + h_j \cos(2u/m)] \sin \phi_2 \\ -[R_2 + h_j \cos(2u/m)] \cos \phi_2 + (u - R_2 \phi_2) \sin \phi_2 \end{bmatrix}$$  \hspace{1cm} (9)

The relative velocity between the generating rack cutter and the generated gear $G_2$ is as follows:

$$V^{(r_2)}(u, \phi_2) = V^{(r_2)}_x i_x + V^{(r_2)}_y j_y + V^{(r_2)}_z k_z = \omega_2 h_j \cos(2u/m) i_x + \omega_2(u - R_2 \phi_2) j_y + 0 k_z.$$  \hspace{1cm} (10)

where $\omega_2$ is angular velocity of gear $G_2$. According to the theory of gearing, the necessary condition for the existence of envelope to the family of surfaces represented by Eq. (10) is the following equation of meshing:

$$\Phi_2(u, \phi_2) = N_r(u) \cdot V^{(r_2)}(u, \phi_2) = \omega_2 \left( R_2 \phi_2 - u + \frac{h_j^2 u \sin(4u/m)}{m} \right) = 0$$  \hspace{1cm} (11)

As $\omega_2$ is not equal to zero, the parameter $\phi_2$ in Eq. (11) can be solved out as a function of $u$ as follows:

$$\phi_2 = \frac{1}{R_2} \left( u - \frac{1}{m} h_j^2 \sin(4u/m) \right)$$  \hspace{1cm} (12)

The mathematical model of tooth geometry of cosine gear $G_2$ can then be represented by

$$r_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} (u - R_2 \phi_2) \cos \phi_2 + [R_2 + h_j \cos(2u/m)] \sin \phi_2 \\ -[R_2 + h_j \cos(2u/m)] \cos \phi_2 + (u - R_2 \phi_2) \sin \phi_2 \end{bmatrix}$$  \hspace{1cm} (13)

### 3. Condition of Undercutting

According to the meshing theory of gearing [14], the proposed cosine gears will be undercut by the cosine rack cutter if the rack cutter has undercutting limit points on its tooth surface. In other words, if the rack cutter does not have any undercutting limit point on its tooth surface, the generated surfaces of cosine gears will not be undercut. Undercutting limit points are also referred to as the limit points of the first kind. Since the two cosine gears are generated by the same rack cutter, they have the same condition of undercutting. Here, the cosine gear $G_1$ is chosen to derive the condition of undercutting. According to the general form of condition of undercutting presented in [14], the undercutting limit points on the surface of the cosine rack cutter are determined by the following condition:
where

\[ E = \left( \frac{\partial r_x}{\partial u} \right), F = \left( \frac{\partial r_y}{\partial v} \right), G = \left( \frac{\partial r_z}{\partial \phi} \right) \]

After substituting Eq. (1) and Eqs. (5)-(7) into Eq. (14) and replacing \( R_t \) and \( h_f \) by the following equations:

\[ R_t = N_t \cdot m / 2; h_f = h_f^* \cdot m \] (15)

the condition of undercutting represented in Eq. (14) can yield the following equations:

\[ N_t = -2h_f^* \cos(2u / m) \left[ -1 + 4(h_f^*)^2 \cos(4u / m) \right] \] (16)

There is an extreme value for \( N_t \) when \( u \) satisfies the following condition:

\[ u = m \cos^{-1} \left( \frac{1}{2} + \frac{1 + 4h_f^*}{4\sqrt{6}h_f^*} \right) \] (17)

After substituting Eq. (17) into Eq. (16), the lower bound for the number of teeth of non-undercutting cosine gear is obtained as follows:

\[ N_{LB} = \frac{1}{3} \sqrt{\frac{2}{3} \left[ 1 + 4(h_f^*)^2 \right]^{3/2}} \] (18)

According to Eq. (18), if the generating cosine rack cutter is provided with a standard tooth height, which means the dedendum coefficient \( h_f^* \) is 1.25, then the lower bound for the number of teeth \( N_{LB} \) will be 5.313. In other words, as long as the number of teeth of the cosine gear is larger than 6, the cosine gear will not be undercut by the cosine rack cutter. Table 1 shows the relationship of \( h_f^* \), \( N_{LB} \), and \( N_{min} \). When \( h_f^* \) is increased from 1.25 to 1.4, \( N_{LB} \) will increase from 5.313 to 7.153. That is to say, if \( h_f^* \) is 1.25, the minimum number of teeth of non-undercutting \( N_{min} \) is 6. If \( h_f^* \) is 1.34, the minimum number of teeth of non-undercutting \( N_{min} \) is 7. If \( h_f^* \) is 1.4, the minimum number of teeth of non-undercutting \( N_{min} \) is 8.

4. Analytic Mathematical Model for Evaluating Contact Ratio

Contact Ratio is a very important performance measure for the measurement of transmission quality of a pair of gears. Contact ratio also determines the share of load between two adjacent teeth. To evaluate contact ratio, the first step is to determine the line of action. As shown in Figure 2, two coordinate systems \( S_1(x_1, y_1, z_1) \) and \( S_2(x_2, y_2, z_2) \) are applied to connect rigidly to the gear housing. By transforming the mathematical model of tooth geometry of the cosine rack cutter from \( S_1(x, y, z) \) to \( S_1(x_1, y_1, z_1) \) and replacing the parameter \( \phi \) by Eq. (7), the mathematical model of the line of action can be represented in \( S_1 \) as follows:

\[ x_1 = \frac{1}{m} h_f^* \sin(4u / m); y_1 = R_t - h_f \cos(2u / m) \] (19)

As shown in Figure 3, the line of action intersects the addendum circle of G1 at point P1. At point P1, the following condition is true:
\( (x_1)^2 + (y_1)^2 - (R_1 + h_1)^2 = 0 \) \hspace{1cm} (20)

where \( h_1 \) is addendum tooth height. Equation (20) is a nonlinear equation with one unknown parameter \( u \). Based on Eq. (20), the analytic solution of \( u \), denoted by \( u_{p1} \), can be expressed as follows:

\[
\begin{align*}
  u_{p1} & = \left( \frac{m}{2} \right) \cos^{-1} \left( K_1 - \frac{1}{2} \sqrt{Q_1 - \frac{2C_1}{A_1K_1}} \right) \\
  A_1 & = -(4h_1^4/m^2); \quad B_1 = h_1^2 - A_1; \quad C_1 = -2h_1R_1; \quad D_1 = -2h_1R_1 - h_1^2; \\
  T_1 & = 2B_1^3 + 27AC_1 - 72AB_1D_1; \quad M_1 = \left( T_1 + \sqrt{-4(B_1^2 + 12A_1D_1)^3 + T_1^2} \right)^{1/3}; \\
  K_1 & = \frac{-2B_1 + 2^{1/3}(B_1^2 + 12A_1D_1)}{3A_1M_1} + \frac{M_1}{3(2^{1/3})A_1}; \\
  Q_1 & = \frac{-4B_1 - 2^{1/3}(B_1^2 + 12A_1D_1)}{3A_1M_1} - \frac{M_1}{3(2^{1/3})A_1};
\end{align*}
\hspace{1cm} (21)

Similarly, the mathematical model of tooth geometry of the cosine rack cutter is transformed from \( S_r(x, y, z) \) to \( S_{r2}(x_2, y_2, z_2) \) and the parameter \( \phi_1 \) is replaced by Eq. (7). Then the line of action can be represented in \( S_{r2} \) by

\[
x_2 = \frac{1}{m} h_2^2 \sin(4u/m); \quad y_2 = -R_2 - h_2 \cos(2u/m) \hspace{1cm} (22)
\]

The line of action intersects the addendum circle of \( G_2 \) at point \( P_2 \). At point \( P_2 \), the following condition is true:

\[
\left( x_2 \right)^2 + \left( y_2 \right)^2 - \left( R_2 + h_2 \right)^2 = 0 \hspace{1cm} (23)
\]

Equation (23) is also a nonlinear equation with one unknown parameter \( u \). The analytic solution of \( u \) is solved based on Eq. (23) and is denoted by \( u_{p2} \) as follows:

\[
\begin{align*}
  u_{p2} & = \left( \frac{m}{2} \right) \cos^{-1} \left( K_2 - \frac{1}{2} \sqrt{Q_2 - \frac{2C_2}{A_2K_2}} \right) \\
  A_2 & = -(4h_2^4/m^2); \quad B_2 = h_2^2 - A_2; \quad C_2 = 2h_2R_2; \quad D_2 = -2h_2R_2 - h_2^2; \\
  T_2 & = 2B_2^3 + 27AC_2 - 72AB_2D_2; \quad M_2 = \left( T_2 + \sqrt{-4(B_2^2 + 12A_2D_2)^3 + T_2^2} \right)^{1/3}; \\
  K_2 & = \frac{-2B_2 + 2^{1/3}(B_2^2 + 12A_2D_2)}{3A_2M_2} + \frac{M_2}{3(2^{1/3})A_2}; \\
  Q_2 & = \frac{-4B_2 - 2^{1/3}(B_2^2 + 12A_2D_2)}{3A_2M_2} - \frac{M_2}{3(2^{1/3})A_2};
\end{align*}
\hspace{1cm} (24)

With \( u_{p1} \) and \( u_{p2} \), contact ratio, denoted by \( CR \), can then be evaluated by the following equations:

\[
\begin{align*}
  \phi_1^{(P1)} & = \frac{u_{p1} - h_1^2 \sin(4u_{p1}/m)}{R_1}; \quad \phi_2^{(P2)} = \frac{u_{p2} - h_2^2 \sin(4u_{p2}/m)}{R_2}; \quad CR = \frac{\phi_1^{(P1)} - \phi_2^{(P2)}}{2\pi/N_1}
\end{align*}
\hspace{1cm} (25)

5. Numerical Example

A numerical example of a cosine gear drive composed of a pair of cosine gears is presented based on the mathematical model of tooth geometry developed in section 2 of this paper. Figure 4 shows the cosine gear drive which uses the following settings: gear module \( m \) is 5mm, the number of teeth of
pinion gear G1 is 6, the number of teeth of gear G2 is 9, the addendum coefficient $h_a^*$ is 1, the dedendum coefficient $h_f^*$ is 1.25. According to Table 1, the minimum number of teeth for a non-undercutting cosine gear is 6 if $h_f^* = 1.25$. Therefore, it can be observed that the shape of teeth of the pinion gear is smooth. Since the number of teeth of cosine gear drive can be very small, the volume of gearbox can become small as well. When facing the mission to reduce the size of gearbox, designers may consider to replace involute gears by cosine gears since the minimum number of teeth of non-undercutting for cosine gears is smaller than that of involute gears.

6. Cause-and-Effect Analysis for Contact Ratio

According to the analytic mathematical model for evaluating contact ratio proposed in section 4 of this paper, causes that might affect contact ratio include gear module, gear ratio, addendum coefficient, and dedendum coefficient. In this section, the numerical example presented in section 5 of this paper is continued to be used to execute the cause-and-effect analysis for contact ratio.

6.1 Effect of Gear Module

The purpose of studying the effect of gear module on contact ratio herein is to demonstrate that the version of cosine gear drive proposed in this paper is different from that proposed by Luo et al. [1]. According to Yu et al. [2], the contact ratio of the version proposed by Luo et al. is influenced by gear module. However, the contact ratio of the version proposed in this paper is not influenced by gear module. Table 2 shows that when gear module $m$ is increased from 5mm to 20mm, contact ratio ($CR$) remains constant all the time. Contact ratio is not influenced by gear module at all. It should be noted that although gear module does not affect contact ratio, gear module is still an important factor which can affect tooth size and tooth strength of gear.

6.2 Effect of Gear Ratio

Gear ratio is defined as the ratio of $N_2$ to $N_1$, where $N_2$ is the number of teeth of gear G2 and $N_1$ is the number of teeth of gear G1. A higher gear ratio implies a higher capability of speed reduction. As shown in Figure 5, the gear ratio is gradually increased from 1 to 5 while the number of teeth $N_1$ is provided with three fixed levels: 6, 9, and 12. When $N_1$ is 6 and gear ratio is increased from 1 to 5, contact ratio will rise from 1.1318 to 1.1935 with an increment of 5.45%. When $N_1$ is 9 and gear ratio is increased from 1 to 5, contact ratio will rise from 1.1976 to 1.2225 with an increment of 2.94%. When $N_1$ is 12 and gear ratio is also increased from 1 to 5, contact ratio will rise from 1.2166 to 1.2369 with an increment of 1.67%. It can be found that the effect of gear ratio is significant when $N_1$ is small and the effect of gear ratio is not significant when $N_1$ is large.

6.3 Effect of Addendum Coefficient

To study the effect of addendum coefficient on contact ratio, addendum coefficient is varied from 0.8 to 1.2 and the other parameters remain unchanged. The effect of addendum coefficient is shown in Figure 6. Contact ratio will rise initially and then fall as addendum coefficient is increased. Contact ratio has an extreme value of 1.2502 when addendum coefficient is 1.19.

6.4 Effect of Dedendum Coefficient

To study the effect of dedendum coefficient on contact ratio, dedendum coefficient is varied from 1.25 to 1.35 and the other parameters are kept invariant. The effect of dedendum coefficient on contact ratio is shown in Table 3. Contact ratio will rise as dedendum coefficient is increased and will go higher as dedendum coefficient is increased continuously.
7. Conclusion

This paper has proposed a new version of cosine gear drive, which is composed of a pair of cosine gears generated by a cosine rack cutter. A mathematical model of tooth geometry of the cosine gears is developed based on the coordinate transformation theory and the theory of gearing. The condition of undercutting is established based on the meshing theory of gearing is applied to determine the lower bound of number of teeth of non-undercutting cosine gear. The minimum number of teeth of non-undercutting of the proposed cosine gear is only 6. An analytic mathematical model for the analysis of contact ratio is proposed. A cause-and-effect analysis for contact ratio is performed. According to the results of analysis, the following findings can be drawn. First, gear module does not affect contact ratio at all. Second, increasing gear ratio can increase contact ratio. Third, to increase contact ratio, addendum coefficient should be optimized. Forth, increasing dedendum coefficient can increase contact ratio. In summary, to obtain a higher contact ratio, it is suggested to increase the number of teeth, increase gear ratio, optimize addendum coefficient, and increase dedendum coefficient.

8. References

Table 1. Relationship of $h^*_{fL}$, $N_{LB}$, and $N_{min}$

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<th>$h^*_{fL}$</th>
<th>1.25</th>
<th>1.28</th>
<th>1.31</th>
<th>1.34</th>
<th>1.37</th>
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Table 2. Gear module versus contact ratio

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<th>$m$ (mm)</th>
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<th>11</th>
<th>14</th>
<th>17</th>
<th>20</th>
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<td>1.1597</td>
<td>1.1597</td>
<td>1.1597</td>
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Table 3. Dedendum coefficient versus contact ratio.

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<th>$h^*_{fL}$</th>
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<td>1.1816</td>
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<td>1.2011</td>
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Figure 1. Tooth profile of cosine rack cutter

Figure 2. Relative motion of coordinate systems in the process of generation of cosine gears

Figure 3. The line of action intersecting addendum circles at points P1 and P2

Figure 4. A cosine gear drive composed of a pair of cosine gears
Figure 5. Effect of gear ratio on contact ratio

Figure 6. Effect of addendum coefficient on contact ratio