Optimization Model Integrated Flight Schedule and Maintenance Plans

Shao Zhifang, Sun Lu, Li Fujuan

1 School of Information Management and Engineering, Shanghai University of Finance and Economics, Shanghai 200433, China, szhifang@yahoo.com.cn
2 School of Information Management and Engineering, Shanghai University of Finance and Economics, Shanghai 200433, China, narudo_go@126.com
3 China Eastern Airlines Co. Ltd., Shanghai 201202, China, lifj@ceair.com

Abstract

Airline companies have to control their operating costs by managing their aircraft effectively. Analytical techniques have been used to solve such complex problems related to airline operations planning which often be classified into four types: “fleet assignment problems”, “crew scheduling”, “maintenance scheduling”, and “revenue management.” Due to the complexity of the combined optimization problem, most of the studies only address one of them to formulate the optimization models which have less effective to the real world problems. In this paper, we considered flight frequency, fleet assignment and maintenance simultaneously; an integrated planning model has been build and solved to get the maximum profit. The model is validated using the real data from an airline company and the results show it’s effective.

Key Words: Maintenance Plan, Fleet assignment, Optimization, Model

1. Introduction

As one of the primary products of an airline, a flight schedule defines a feasible plan of what cities to fly to and at what times. The task of airline schedule planning is to generate a flight schedule that achieves the most effective use of an airline’s resources. Profitable solutions require anticipation of general market conditions: the costs of capital, fuel and labor, as well as the level and nature of competition [1]. Airlines address many scheduling issues with large-scale combinatorial optimization techniques. The scale of today’s airlines makes this increasingly difficult. This motivates the extensive use of analytical techniques to solve such complex problems related to airline operations planning. In the literature, several modeling and solution approaches have been proposed to individually solve the problems related to the different airline planning processes, which can be classified as assigning aircraft and crews to flights, routing aircraft to maintenance bases. Works in these areas can be find in Gopalan and Talluri [2], Hane et al. [3], and McGill and Van Ryzin [4-5] and so on. Due to the complexity of these problems, they are typically considered in isolation, and only after the flight schedule has been determined. Recently, an increasing effort is being made in order to develop novel approaches for integrating some of the airline operations planning problems such as Cordeau et al. [5], who integrate the aircraft routing and crew scheduling problems for a single fleet type. Sandhu and Klabjan[6] (2007) proposed a model for simultaneously considering fleeting, aircraft routing, and crew scheduling. Ki-Hwan Bae[7] gives three integrated models in his dissertation for Ph.D. degree: (1) integrate fleet assignment and schedule design (2) integrate the schedule design, fleet assignment, and aircraft maintenance routing decisions (3) the crew scheduling problem is additionally integrated with fleet assignment and aircraft routing.

In this paper, we propose a model that considers flight frequency, flight time, fleet assignment and maintenance simultaneously. The model is validated using real world data. All the work is organized into five sections: section 2 is problem description. The integrated model is presented in Section 3. Section 4 outlines solution methodologies and results. Finally, in Section 5 we conclude the work with a brief review of the optimization model.
2. Problem Description

The formulation of flight schedule and associated plans is one of the most important works for an airline company. Flight schedule need to assign a specific aircraft to an air route while some conditions must considered such as the aircraft’s type, passenger flow volume, rules of Civil Aviation Administration of China (CAAC) and so on. At the same time, aircraft must meet airline requirements for scheduled maintenance. Aircraft must undergo four types of maintenance, commonly one of the following: A check, B check, C check, or D check. In this paper, B check is considered here for A check need little time which will not affect the schedule, while C and D check, which time interval exceed the schedule time bucket in our work.

We have some hypothesis as following:
- The airline network has been determined and the fight schedule and maintenance plan are all based on this airline network.
- The passenger flow volume can be forecast and has little tolerance to the fact.
- Maintenance needs one whole day to finish, that is, if an aircraft is scheduled to fly on a day, it cannot be scheduled to maintenance, vice versa.

3. Model Formulation

3.1 Notation:

- $\alpha$: Aircraft in an airline company;
- $N$: set of aircrafts in an airline company, $\alpha \in N = \{1,2,3...n\}$.
- $rs$: an air route in airline network of a company.
- $r$: departure airport of an air route.
- $s$: arrival airport of an air route.
- $A$: airline network of an airline company.
- $Q$: set of airports.
- $J$: set of airports that aircrafts landing.
- $E(r_s)$: expected price for r-s line.
- $E(l, l)$: expected seat utilization rate for aircraft $\alpha$ fly $r-s$ line.
- $E\epsilon e$: expected number of spilled passengers for aircraft $\alpha$ flying $r-s$ route, which is a positive integer.
- $P_{\alpha}$: standard sit capacity of aircraft $\alpha$, which is determined by the aircraft type.
- $D_{\alpha,s}$: Passenger traffic in r-s route in the $\beta$th day.
- $U_{\beta}$: The total available hours of the aircrafts in the $\beta$th day.
- $CO_{\alpha,s}$: Operation cost of aircraft $\alpha$ in a single flight when flying r-s route.
- $C_i$: the unit cost of spilled passengers.
- $C_{\alpha}$: the cost of aircraft $\alpha$ fly one hour.
- $t_{\alpha,s}$: time spend of aircraft $\alpha$ on flying r-s route.
- $U_{\beta}$: the total flying hours of an airline company in the $\beta$th day.
- $\phi$: is the maximum number of aircraft that can take-off at airport $r$ in a day.
- $\mu$: the maximum number of aircraft that can land at airport $s$.
- $\beta$: the starting date for maintenance, here, T is the maintenance planning cycle, $T = \{1,2,...,m\}$.
- $C_{\alpha,\beta}$: the AOG(Aircraft on Ground) loss for aircraft $\alpha$ maintained in the $\beta$th day.

$x_{\alpha,\beta}$ and $x_{\alpha,\beta}$ are decision variables, they are binary variables.

$$x_{\alpha,\beta} = \begin{cases} 1 & \text{if aircraft } \alpha \text{ maintained at the } \beta \text{th day} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{\alpha,\beta} = \begin{cases} 1 & \text{if aircraft } \alpha \text{ fly r-s line at the } \beta \text{th day} \\ 0 & \text{otherwise} \end{cases}$$

$E_{\alpha,\beta}$ is the earliest (latest) maintenance time allowed for aircraft $\alpha$.

$H_{\beta}$ is maximum number of aircraft that can be maintained at $\beta$ day.

$r_{\alpha,\beta}$ is the maintenance resources needed by aircraft $\alpha$ maintained at $\beta$th day.
$R_\beta$: the maximum resources can be offered at the $\beta^{th}$ day.

$g_{\alpha,\beta}$: the work hour for aircraft $\alpha$ maintenance at the $\beta^{th}$ day.

$G_\beta$: the maximum work hour that the maintenance base can be offered at the $\beta^{th}$ day.

$F_\beta$: is the number of aircrafts that will be needed at $\beta^{th}$ day, which is the number of flight we can schedule at $\beta^{th}$ day.

### 3.2 Model construction

The objective of this model is to get the maximum profit. For an airline company, the revenue mainly comes from tickets sale, while the cost involve operation cost, the loss from spilled passenger and AOG loss. So, the model can be formulated as following:

\[
\text{Max } \pi = \sum_{\beta=1}^{n} \sum_{\alpha=1}^{m} \left( E(f_{\alpha,\beta}) \cdot P_{\alpha} \cdot E(l_{\alpha,\beta}) \cdot x_{\alpha,\beta,\epsilon} - C_t \cdot x_{\alpha,\beta,\epsilon} - CO_{\alpha,\beta} \cdot x_{\alpha,\beta,\epsilon} - E(e_{\alpha,\beta}) \cdot C_e \right),
\]

\[
\text{S.t.} \quad \sum_{\beta=1}^{n} P_{\alpha} \cdot x_{\alpha,\beta,\epsilon} \geq D_{\beta,\epsilon}, \forall \beta \in T, rs \in A
\]

\[
CO_{\alpha,\beta,\epsilon} = CE_{\alpha,\epsilon} \cdot t_{\alpha,\epsilon}, \sum_{\alpha=1}^{m} \sum_{s \in A} t_{\alpha,\epsilon} \cdot x_{\alpha,\beta,\epsilon} \leq U_{\beta}, \forall \beta \in T
\]

\[
\sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon} > 0, \forall \beta \in T, rs \in A
\]

\[
\sum_{\epsilon \in E} \sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon} \leq \varphi_{\epsilon}, \forall \beta \in T, r \in Q
\]

\[
\sum_{r \in Q} \sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon} \leq \mu_{\epsilon}, \forall \beta \in T, s \in J
\]

\[
\sum_{\epsilon \in E} \sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon} = \sum_{\epsilon \in E} \sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon}, \forall \beta \in T
\]

\[
\sum_{\epsilon \in \epsilon} \sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon} \leq 1, \forall \beta \in T, \alpha \in N
\]

\[
x_{\alpha,\beta,\epsilon} \cdot x_{\alpha,\beta} = 0, \forall \beta \in T, \alpha \in N, rs \in A
\]

\[
E_{\epsilon} \leq \beta \leq l_{\alpha,\epsilon}, \forall x_{\alpha,\beta} = 1, \alpha \in N
\]

\[
\sum_{\alpha=1}^{m} x_{\alpha,\beta,\epsilon} \leq H_{\beta}, \forall \beta \in T
\]

\[
\sum_{\alpha=1}^{m} g_{\alpha,\beta} \leq R_{\beta}, \forall \beta \in T
\]

\[
\sum_{\alpha=1}^{m} g_{\alpha,\beta} \leq G_{\beta}, \forall \beta \in T
\]

\[
\sum_{\alpha=1}^{m} (1 - x_{\alpha,\beta}) \geq F_{\beta}, \forall \beta \in T
\]

The objective function maximizes the net profit which consists of four parts: revenue from tickets sale, operation cost, loss from spilled passenger and loss from AOG (Aircraft on Ground). Equations (2) to (14) are the constraints explained as following:
Constraint (2) is the capacity limit, which guarantees that the number of seats offered by each aircraft route in a day not less than the passenger’s requirement.

Equation (3) limit the total flying hours. The fly time for each fleet type scheduled cannot exceed the total fly time available.

Equation (4) represents the cover constraints that stipulate each air route must be assigned a fleet type.

Constraint (5) requires the number of takeoff aircrafts must in the scope of allowed take off number. While constraint (6) required the number of landing aircrafts must in the scope of allowed landing number.

Equation (7) is the balance constraint, which stipulates that the number of arrivals must equal the number of departures for each station, time event and fleet type.

Equation (8) describes that if an aircraft is assigned, it only can be assigned one time in the same day.

Constraints (9) prescribes that if an aircraft fly in a day, it must not maintained in the same day and vice versa.

Constraint (10) limit aircraft must maintain in allowed time range.

While constrain (11) required the number of aircrafts in maintenance less than the capacity of maintenance base.

The resources needed for maintenance less than the available resource in the maintenance base is described by equation (12).

Constraints (13) required the work hours needed by aircrafts in a day less than the work hours can be offered by maintenance base.

The last constraint required except the aircrafts in maintenance, the other aircrafts can offer enough flight hours to match the schedule.

4. Computational Results and Discussion

The flight schedule and maintenance model is a large-scale Mix Integer Nonlinear Programming model and it can be solved using the commercially available LP/IP solver CPLEX. We test the model and algorithm on actual large-scale data set of a major China airline.

To illustrate the approaches, we use a 7-city example which consist of 7 stations (cities) and 6 round-trip air routes involve Shanghai-Beijing (SH-BJ), Shanghai-Shenzhen (SH-SZ), Shanghai-Xiamen(SH-XM), Shanghai-Tianjin(SH-TJ), Shanghai-Qingdao(SH-QD) and Shanghai-Guangzhou(SH-GZ). There are 18 aircrafts for this network, involve 2 big aircraft B747-400 which capacity is 400. The small aircraft can hold 150 persons. All the data about the aircrafts can be found in table1. Table 2 shows us the passenger flow volume forecast in the fifteen days in the future. From table 2 we can see that the air route SH-BJ and SH-TJ are busy routes while SH-SZ and SH-QD have fewer passengers. Table 3 is the expected price.

The results from the model can be found in Table 4 and table 5, which show us the aircraft schedule and maintenance schedule respectively. With this schedule, we can get profit 2718706 Yuan in 15 days, much higher than the data from the company. In table 4, Ni (i=1,2……,18) is the aircraft Number. From table4, we can see that in the first day, aircraft 10 and aircraft 12 will fly the air route Shanghai-Beijing, while air route Shanghai-Shenzhen will be covered by aircraft 11, aircraft 14 and aircraft 17 and so as the other air routes. Table 2 is the maintenance date for each aircraft, such as aircraft 1 will be maintenance on the third day, and aircraft 2 will be maintenance on the fifth day and so on. Figure 1 illustrates aircraft NO.10’s schedule, 6 days in the schedule NO.10 fly SH-BJ air route and 5 days fly SH-TJ air route, the other few days it fly SH-SZ, SH-GZ air route, as we know that NO.10 is a large aircraft can hold 400 person and SH-BJ, SH-TJ are two busy routes, the results fit this fact better. So as the aircraft NO.7, which has small capacity, so most of the days it fly SH-QD, SH-XM air route which have fewer passenger, as fig 2 shows.
Table 1. Aircrafts data for illustrative example

<table>
<thead>
<tr>
<th>Aircrafts available (aircrafts NO.)</th>
<th>Capacity</th>
<th>mean fuel consumption in flight (T/Hour)</th>
<th>Cost (Yuan/ Hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B737-700 8(NO.1-8)</td>
<td>150</td>
<td>2.5</td>
<td>11250</td>
</tr>
<tr>
<td>B747-400 2(NO.9-10)</td>
<td>400</td>
<td>10.8</td>
<td>48600</td>
</tr>
<tr>
<td>B757-200 3(NO.11-13)</td>
<td>200</td>
<td>3.8</td>
<td>17100</td>
</tr>
<tr>
<td>A320 5(NO.14-18)</td>
<td>180</td>
<td>3.0</td>
<td>13500</td>
</tr>
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</table>

Table 2. Passenger flow volume forecast (Unit: Person)

<table>
<thead>
<tr>
<th>Day</th>
<th>SH-BJ</th>
<th>SH-SZ</th>
<th>SH-XM</th>
<th>SH-TJ</th>
<th>SH-GZ</th>
<th>SH-QD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>540</td>
<td>314</td>
<td>350</td>
<td>420</td>
<td>408</td>
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<tr>
<td>2</td>
<td>540</td>
<td>314</td>
<td>350</td>
<td>420</td>
<td>408</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>540</td>
<td>314</td>
<td>350</td>
<td>420</td>
<td>408</td>
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<td>314</td>
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<td>420</td>
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<td>300</td>
</tr>
<tr>
<td>6</td>
<td>600</td>
<td>374</td>
<td>410</td>
<td>480</td>
<td>468</td>
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<td>7</td>
<td>600</td>
<td>374</td>
<td>410</td>
<td>480</td>
<td>468</td>
<td>360</td>
</tr>
<tr>
<td>8</td>
<td>600</td>
<td>374</td>
<td>410</td>
<td>480</td>
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<td>374</td>
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<td>480</td>
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<td>360</td>
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<tr>
<td>11</td>
<td>570</td>
<td>344</td>
<td>380</td>
<td>450</td>
<td>438</td>
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<tr>
<td>12</td>
<td>570</td>
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<td>380</td>
<td>450</td>
<td>438</td>
<td>330</td>
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<td>15</td>
<td>570</td>
<td>344</td>
<td>380</td>
<td>450</td>
<td>438</td>
<td>330</td>
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Table 3. The expected price

<table>
<thead>
<tr>
<th>Air route</th>
<th>Expected price (Yuan)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH-BJ</td>
<td>800</td>
</tr>
<tr>
<td>SH-SZ</td>
<td>1000</td>
</tr>
<tr>
<td>SH-XM</td>
<td>600</td>
</tr>
<tr>
<td>SH-TJ</td>
<td>800</td>
</tr>
<tr>
<td>SH-GZ</td>
<td>1000</td>
</tr>
<tr>
<td>SH-QD</td>
<td>600</td>
</tr>
</tbody>
</table>
Table 4. Aircrafts schedule

<table>
<thead>
<tr>
<th>Day</th>
<th>SH-BJ</th>
<th>SH-SZ</th>
<th>SH-XM</th>
<th>SH-TJ</th>
<th>SH-GZ</th>
<th>SH-QD</th>
</tr>
</thead>
</table>

Table 5. Maintenance schedule

<table>
<thead>
<tr>
<th>Aircraft No.</th>
<th>Maintenance day</th>
<th>Aircraft No.</th>
<th>Maintenance day</th>
<th>Aircraft No.</th>
<th>Maintenance day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D3</td>
<td>7</td>
<td>D13</td>
<td>13</td>
<td>D7</td>
</tr>
<tr>
<td>2</td>
<td>D5</td>
<td>8</td>
<td>D9</td>
<td>14</td>
<td>D6</td>
</tr>
<tr>
<td>3</td>
<td>D5</td>
<td>9</td>
<td>D2</td>
<td>15</td>
<td>D3</td>
</tr>
<tr>
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<td>D7</td>
<td>10</td>
<td>D14</td>
<td>16</td>
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<tr>
<td>5</td>
<td>D11</td>
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<td>D10</td>
<td>17</td>
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</tr>
<tr>
<td>6</td>
<td>D3</td>
<td>12</td>
<td>D9</td>
<td>18</td>
<td>D11</td>
</tr>
</tbody>
</table>

Fig 1. Schedule for NO.10 aircraft

Fig 2. Schedule for NO.7 aircraft

5. Conclusions

In a highly competitive environment, airline companies have to control their operating costs by managing their aircraft effectively. The flight schedule is not only a complex work which refers to a multi-constrained and combinatorial optimization problem, but also the important issue for an airline company must deal with in the real production and operation. In this paper, the objective function is to maximum the profit, we considered the revenue from tickets sale, the operation cost, the spill cost and
the maintenance cost. While the constraints involve plane count constraint, balance constraint, cover constraint, maintenance constraint and airport constraints and so on. We proposed an integrated model considered flight frequency, flight time, fleet assignment and maintenance simultaneously, an actual 7 stations data set from an airline is addressed to test the model and the results show it is effective.

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7. References