Experimental Investigation into Honeycomb Paperboard Vibration Transmissibility Property and Nonlinear Parameters Identification

Dapeng Zhu, Ruichun He, Guo Wang

Abstract
The vibration transmissibility property of honeycomb paperboard is investigated by experiments, the experimental results indicate that both the resonance frequency and resonance peak are sensitive to the altitude of excitation force. We can assume that both the damping and stiffness properties are nonlinear, and they can be expressed as the combination of the linear part with the cubic nonlinear part. An experimental system are formulated to record the experimental data, a system parameter identification procedure is presented base on harmonic balance method and total least square method. The system parameters under different static load and different excitation force amplitude conditions are identified and presented in this paper, the system model along with the identified can be used to simulate Tr-f curves, the comparison between the simulated curves with experimental data indicates the model is accurate to predict the vibration transmissibility property, which is important for proper use of honeycomb paperboard and anti-vibration packaging design.

Keywords: Honeycomb Paperboard, Vibration Transmissibility, Experimental Investigation, Harmonic Balance Method, Total Least Square Method

1. Introduction
Honeycomb paperboard is a kind of sandwich panel, it is made up of three parts: the upper and lower liners, between which is the honeycomb core. All parts are made of reusable paper. Because of its specific structure, it has many advantages over other materials, such as the high strength-to-weight ratio, high stiffness-to-weight ratio, light weight, the ease to be processed, and it is recyclable, reusable and biodegradable. Because of these advantages, honeycomb paperboard has been used in many fields. In recent years, because of the environment protection concerns and the command for reducing the plastic wastes, people began to use honeycomb paperboard in protective packaging as the cushion material to substitute the foam. The realization of the potential of honeycomb paperboard as an important cushion material has inspired a close scrutiny of its properties.

Guo and Zhang investigated the cushion properties and vibration transmissibility properties of honeycomb paperboard with different thickness by a series of experiments, the experiment results are fitted by polynomials, the results provided basic data for protective packaging design[1,2]. Wang investigated the cushioning properties of honeycomb paperboard by experimental analysis[3]. The impact behavior and energy absorption properties of honeycomb paperboard were presented in [4], the experimental results indicated that the increase of the relative density of paper honeycomb cores can efficiently improve the dynamic cushioning properties. The critical buckling load of honeycomb paperboard under out-of-plane pressure was investigated by analyzing the structure and the collapse mechanism [5], the models and the calculation method in the paper can be used to predict the static critic buckling load. Zhu[6,7] modeled honeycomb paperboard as a linear material with viscoelastic property, the identified parameters can be used to predict the forced and free response of honeycomb paperboard-mass system. Because the main component of honeycomb paperboard is paper, the relative humidity will affect its properties greatly, in [8,9], mathematical model was developed to describe the relationship between the energy absorption properties of paper honeycombs and ambient humidity, as well as the structural parameters. Compression tests and plate shear tests were carried out to investigate the compressive and shear properties of honeycomb paperboard, the influence of moisture on the material and its behavior under sustained loads were investigated in [10,11], which provided designers
and engineers with knowledge of the material properties and its relative advantages and disadvantages in comparison to other materials.

The research above provided the basic knowledge of honeycomb paperboard properties, but there is few reference about the vibration transmissibility property of honeycomb paperboard. The vibration transmissibility \( T \) is defined as \[
T = \frac{\ddot{x}}{\ddot{y}}
\] (1)

Where \( T \) is the acceleration transmissibility \( \ddot{x} \) and \( \ddot{y} \) are the acceleration output and input of the packaging system respectively, \( \ddot{y} \) denotes the amplitude of the vibration data. The acceleration transmissibility is usually the function of vibration frequency \( f \), i.e. the vibration transmissibility properties of the honeycomb paperboard can be expressed by the \( T_f \)-\( f \) curves, and the \( T_f \)-\( f \) curves are important for the packaging design. This paper presents experimental, analytical investigations to examine the characteristics of honeycomb paperboard transmissibility properties in uni-directional deformation. In particular, the effects of excitation amplitude on the transmissibility properties are investigated. A phenomenological modeling of the honeycomb paperboard mechanical properties is developed and used in the governing equation of motion of the honeycomb paperboard isolator. The differential equation of motion includes the stiffness nonlinearity and the damping nonlinearity. The parameters in the equation are identified by use of harmonic balance method. This model can be used to predict the isolator dynamic characteristics and its transmissibility for the excitation vibration with different amplitude.

2. Response analysis of honeycomb paperboard-mass system

Because of the considerably large stiffness of honeycomb paperboard, the amount of the compression will be very small under some load, thus, in many cases, researchers regarded honeycomb paperboard as a linear material[1,2,6,7]. Nevertheless, Nonlinear characteristics in compression deformation exists in mechanical properties such as stiffness and damping. Furthermore, even when the excitation amplitude is small the response amplitude may often be large enough that nonlinearities cannot be ignored, especially at the resonance frequency.

If the honeycomb paperboard stiffness and damping nonlinearity are taken into account, the motion equation of honeycomb paperboard-mass system can be expressed as

\[
m\dddot{z} + c_3\dddot{z} + k_3z + k\ddot{z} = -m\dddot{y}
\] (2)

Where \( m \) is the inertia of the mass, \( c \) and \( k \) are damping and stiffness coefficients respectively, \( c_3 \) and \( k_3 \) are cubic damping and stiffness nonlinearity coefficients respectively, \( z \) is the honeycomb paperboard deformation, \( \dddot{y} \) represents the system excitation acceleration.

To learn the dynamic properties of the mass loaded cushion material system, the researchers usually use random noise or a continuous frequency sweep as the input excitation. There are particular deficiencies with both methods when applied to nonlinear system identification. The use of random input and traditional frequency response estimation procedures using cross and power spectral densities[12] produces the best linear approximation to the system corresponding to this particular input, and therefore will not exhibit a nonlinear shape. Alternatively, the response to a continuous frequency sweep is often used for generating the system frequency response. Although it does not inherently assume system linearity, there are problems associated with this method. In particular, the sweep rate can have large effects on the measured response, this is true even for linear system. Faster sweep smooth out many of the features of the system. A very slow sweep rate will alleviate this problem to a certain degree, but unless careful precautions are taken, the measurement of fine detail of the steady state response is not assured, thus making identification more difficult.

As described above, the measurement technique used in this paper consists of discrete frequency stepping, where the system is vibrated at a single frequency until the steady state condition is achieved. Then the amplitude of the response and the phase change between the acceleration response and the specimen deformation at the excitation frequency are measured. The frequency is then increased to the next value, and the process repeated to generate \( T_f \)-\( f \) curves.
It is hard to find the closed form solutions for Equation (2). However, for a harmonic input, approximate periodic solutions can be constructed by using the method of harmonic balance [13,14]. In this method, it is assumed that the system has a periodic solution of appropriate period. The Fourier series coefficients for the approximate periodic solution are obtained by the usual method of substituting the solution into the differential equation and equating coefficients of like terms. The terms in the Fourier series, and their total number, are chosen based on the nature of the nonlinearity, the excitation frequency, and the desired level of accuracy.

According to Equation (2), we can assume the excitation force

\[ f(t) = -m \ddot{y} = G \cos(\omega t) \]

where \( \omega \) is the circular frequency of the excitation force. Since the equation of motion contains cubic order nonlinearities, the periodic response is expected to contain only odd harmonics of the driving frequency. It is assumed that, at the levels of excitation considered, only the first and third harmonics are more dominant than the higher harmonics. Hence, a reasonable approximation to the solution \( z(t) \) can be written as

\[ z = A e^{i\omega t} + \bar{A} e^{-i\omega t} + B e^{3i\omega t} + \bar{B} e^{-3i\omega t} \] (3)

Where \( A = A_r + j A_i \), \( B = B_r + j B_i \), are the complex Fourier coefficients, \( \bar{A} \) and \( \bar{B} \) are complex conjugate of \( A \) and \( B \) respectively. Equation (3) is substituted into (2), and the coefficients of \( e^{i\omega t} \) and \( e^{3i\omega t} \) are equated. Equating the coefficients of \( e^{i\omega t} \) yields

\[ -m \omega^2 A + j \omega A c + j(3A | A|^2 + 9B \bar{B} + 54A | B|^2) \omega^3 c_3 + A k \]
\[ + (3A | A|^2 + 6A | B|^2 + 3B \bar{B}^2) k_3 = G/2 \] (4)

Similarly, equating the coefficients of \( e^{3i\omega t} \) yields

\[ -9m \omega^2 B + 3j \omega B c + j(-A^3 + 81B | B|^2 + 18B | A|^2) \omega^3 c_3 \]
\[ + B k + (A^3 + 6B | A|^2 + 3B | B|^2) k_3 = 0 \] (5)

By writing \( A = A_r + j A_i \), \( B = B_r + j B_i \), separating the above two equations into their real and imaginary parts, the following equations are obtained. The real part of (4) is

\[ -m \omega^2 A_r - c A_i + (-3A | A|^2 - 9B_r (A_r^2 - A_i^2) - 54A_r | B|^2 + 18A | A|^2) c_3 \]
\[ + kA_r + [3A | A|^2 + 6A_r | B|^2 + 3B_r (A_r^2 - A_i^2)] k_3 = -G/2 = 0 \] (6)

The imaginary part of (4) is

\[ -m \omega^2 A_i + c A_r + [3A | A|^2 + 9B_r (A_r^2 - A_i^2)] + 54A_r | B|^2 + 18A | A|^2 c_3 \]
\[ + kA_i + [3A | A|^2 + 6A_r | B|^2 + 3B_r (A_r^2 - A_i^2)] - 6B_r A_r k_3 = 0 \] (7)

The real part of (5) is

\[ -9m \omega^2 B_r - 3c A_r + (-A_r^3 + 3A_i^2 A_r - 81B_r | B|^2 - 18B_r | A|^2) c_3 \]
\[ + kB_r + [A_r (A_r^2 - A_i^2) - 2A_i A_r + 6B_r | A|^2 + 3B_r | B|^2] k_3 = 0 \] (8)

The imaginary part of (5) is

\[ -9m \omega^2 B_i + 3c A_r + (-A_r^3 + 3A_i^2 A_r + 81B_r | B|^2 + 18B_r | A|^2) c_3 \]
\[ + kB_i + [A_r (A_r^2 - A_i^2) + 2A_i A_r + 6B_r | A|^2 + 3B_r | B|^2] k_3 = 0 \] (9)

Equations (6)–(9) are used in two ways. First, when the system parameters and excitation characteristics are known, these can be solved simultaneously to yield the complex response amplitudes, i.e., \( A_r, A_i, B_r \) and \( B_i \). Alternatively, when the excitation and complex response amplitudes...
are known through measurements or simulations, the equations may be solved to yield the system parameters. In present work, these equations are used to identify the system parameters by the experiment data.

3. Experimental investigation

The vibration transmissibility properties of honeycomb paperboard are investigated by the experiment system shown in Figure 1. This experiment system is made up of three parts: honeycomb paperboard-mass system, shaker, data acquisition system. The specimens are cut to be 250×250mm², the basic weight of the liners is 300g/m², the honeycomb core is made of reusable paper with basic weight of 150g/m². The honeycomb paperboard specimen is glued to thin metal plates which are in turn bolted to the moving base and the rigid mass to prevent them from losing contact with each other during vibration. Before the tests, all the test specimens are preprocessed for 24 hours at temperature 21℃, and relatively humidity 44%.

Two accelerometers are used to record the system acceleration input and output. The high frequency noise in all the signals is cut off by using a low-pass filter. The cut off frequency of the filter was set at 3kHz. The acceleration signals, along with displacement signal, are digitized by a YE6230B dynamic data acquisition equipment (16 bit) and transferred to a PC. All the data record process in this work is under the control of the YE7600 software package. We set the sampling frequency to be 50000 samples per second to calculate the phase change $\phi$ between system acceleration response and specimen deformation accurately.

**Figure 1. Schematic of the setup of honeycomb paperboard vibration transmissibility experiment**

The rigid mass and honeycomb paperboard specimen formulate a mass-material system in Figure 1, this system is used to simulate a package. As with any nonlinear system, the control of input frequency and amplitude is of critical importance to the measured results. Before the measurements, the system is excited at a constant acceleration level over various frequencies for 1 hours, this amount of excitation allowed the honeycomb paperboard specimen to come to its ultimate equilibrium compression level. Then the mass-material system is axially excited with a constant sinusoidal acceleration level by the shaker table at frequency $f = \omega/2\pi$. The acceleration response $\dot{x}$ and the excitation acceleration $\dot{y}$ of the system are measured by accelerometers, the vibration transmissibility of the system $T_r$ at this frequency is obtained by use of Equation (1). The input frequency is then slowly increased to the next value and repeat the measurement process. In regions sufficiently far away from resonance, the step in frequency is 5Hz. However, in regions near the resonance, the frequency step is reduced to 0.5Hz in order to accurately capture the data at the resonance frequency. After this measurement process, we can obtain a steady state $T_r$-$f$ curve to represent the vibration transmissibility properties of honeycomb paperboard.

Because of the nonlinearity of honeycomb paperboard, the vibration transmissibility properties of honeycomb paperboard are influenced by the excitation amplitude. Figure 2 shows the measured $T_r$-$f$ curves for different values of excitation acceleration amplitude $|\dot{y}|$ under different mass weight conditions. Because of the damping and stiffness nonlinearities, under different load and different excitation amplitudes conditions, the honeycomb paperboard transmissibility properties are quite
different, which can be observed in Figure 2. When \( m = 5 \text{kg} \), with the increase of the excitation amplitude from 9.8 m/s\(^2\) to 39.2 m/s\(^2\), the resonance frequency shifts to the left, from 187 Hz to 185 Hz, the peak resonance amplitudes are reduced from 5.3 to 4.1. When \( m = 10 \text{kg} \), with the increase of \( |\ddot{y}| \) from 9.8 m/s\(^2\) to 39.2 m/s\(^2\), the resonance frequency reduced from 133 Hz to 129 Hz, the peak transmissibility reduced from 6.6 to 4.5. While \( m = 15 \text{kg} \), the resonance frequency reduced from 108 Hz to 100 Hz, the peak transmissibility reduced from 7.3 to 4.5.

4. parameters identification

From the data recorded in the experiments, the system parameters can be identified by the procedures outlined below:

Step 1. Record experimental data

The experimental data are recorded by the system shown in Figure 1, assuming the excitation frequency is \( \omega_i (i = 1, 2, \ldots, N) \), at this frequency condition, record the acceleration data of the excitation and response, i.e. record time series of \( \ddot{x} \) and \( \ddot{y} \). Because the precondition of (6)–(9) is that the excitation force of the system \( f(t) = G \cos(\omega t) \), in this paper, the time series \( \ddot{y} \) which have the same phase with \(-\cos(\omega_i t)\) should be chosen, the length of the time series should be chosen to be the exact multiples of \( 2\pi/\omega_i \), this effectively eliminates windowing effects. Time series of \( \ddot{z} \) can be obtained by

\[
\ddot{z} = \ddot{x} - \ddot{y}
\]  

A typical example of the recorded experimental data is shown Figure 3.

Figure 2. Measured \( T_r f \) data of honeycomb paperboard for different excitation amplitudes
(a) \( m = 5 \text{kg} \) (b) \( m = 10 \text{kg} \) (c) \( m = 15 \text{kg} \)

The experimental data are recorded by the system shown in Figure 1, assuming the excitation frequency is \( \omega_i (i = 1, 2, \ldots, N) \), at this frequency condition, record the acceleration data of the excitation and response, i.e. record time series of \( \ddot{x} \) and \( \ddot{y} \). Because the precondition of (6)–(9) is that the excitation force of the system \( f(t) = G \cos(\omega t) \), in this paper, the time series \( \ddot{y} \) which have the same phase with \(-\cos(\omega_i t)\) should be chosen, the length of the time series should be chosen to be the exact multiples of \( 2\pi/\omega_i \), this effectively eliminates windowing effects. Time series of \( \ddot{z} \) can be obtained by

\[
\ddot{z} = \ddot{x} - \ddot{y}
\]  

A typical example of the recorded experimental data is shown Figure 3.
Step 2 Calculate Fourier coefficient of $z$

Similar with (3), time series $z$ can be written in the following form

$$
\tilde{z} = A_e e^{j\omega t} + A_i e^{-j\omega t} + B_e e^{3j\omega t} + B_i e^{-3j\omega t}
$$

The Fourier coefficients $A_e$ and $B_e$ can be obtained by

$$
A_e = \frac{\omega}{2\pi} \int_0^{2\pi} z(k\Delta t)e^{-j\omega k\Delta t} dt = \Delta t \frac{\omega}{2\pi} \sum_{n=0}^{N-1} \tilde{z}(k\Delta t) e^{-j\omega k\Delta t}
$$

$$
B_e = \frac{\omega}{2\pi} \int_0^{2\pi} z(k\Delta t)e^{3j\omega k\Delta t} dt = \Delta t \frac{\omega}{2\pi} \sum_{n=0}^{N-1} \tilde{z}(k\Delta t) e^{3j\omega k\Delta t}
$$

Then, the coefficients $A$ and $B$ in (3) can be obtained by

$$
A = A_e / (-\omega^2) \quad B = B_e / (-9\omega^2)
$$

Step 3 Model parameters estimation

Let Equations (6), (7), (8) and (9) be represented symbolically as $F_n(\omega, m, G_j) (n=1,2,3,4)$, where $\omega_i$ is different excitation circular frequency, $m_i$ is different static load $j=1,2,3$, $m_1=5$ kg, $m_2=10$ kg, $m_3=15$ kg, $G_j(j=1,2,3,4)$ is the amplitude of the different excitation force, and $G_j=mg$, $g_1=9.8$ m/s$^2$, $g_2=19.6$ m/s$^2$, $g_3=29.4$ m/s$^2$, $g_4=39.2$ m/s$^2$. Under different static load condition and different excitation force conditions, according to the recorded experimental data, calculate the Fourier coefficients $A$ and $B$ of system response $z$, substitute $A$ and $B$ into Equation (6)-(9), then the cost function minimized for the estimation of the parameters is

$$
\varepsilon = \sum_{n=1}^{N} \sum_{j=1}^{4} [F_n(\omega, m, G_j)]^2
$$

(11)

Minimizing cost function $\varepsilon$ in (11) by the use of total least square method[16], one can obtain the system parameters in (2) by

$$
Ax = b
$$

(12)

Where

$$
x = [c, c_j, k, k_j]^T
$$

$$
b = \begin{bmatrix}
\sum_{n=1}^{4} \sum_{i=1}^{N} P_{n0} P_{ai} & \sum_{n=1}^{4} \sum_{i=1}^{N} P_{n0} P_{a2} & \sum_{n=1}^{4} \sum_{i=1}^{N} P_{n0} P_{a3} & \sum_{n=1}^{4} \sum_{i=1}^{N} P_{n0} P_{a4}
\end{bmatrix}^T
$$
Experimental Investigation into Honeycomb Paperboard Vibration Transmissibility Property and Nonlinear Parameters Identification

Dapeng Zhu, Ruichun He, Guo Wang

The parameters $P$ in matrix $A$ and $b$ can be obtained by

$$A = \begin{bmatrix}
\sum_{i=1}^{N} P_{11}^i + \sum_{i=1}^{N} P_{12}^i + \sum_{i=1}^{N} P_{13}^i + \sum_{i=1}^{N} P_{14}^i,
\sum_{i=1}^{N} P_{21}^i + \sum_{i=1}^{N} P_{22}^i + \sum_{i=1}^{N} P_{23}^i + \sum_{i=1}^{N} P_{24}^i,
\sum_{i=1}^{N} P_{31}^i + \sum_{i=1}^{N} P_{32}^i + \sum_{i=1}^{N} P_{33}^i + \sum_{i=1}^{N} P_{34}^i,
\sum_{i=1}^{N} P_{41}^i + \sum_{i=1}^{N} P_{42}^i + \sum_{i=1}^{N} P_{43}^i + \sum_{i=1}^{N} P_{44}^i
\end{bmatrix}
$$

The parameters $P$ in matrix $A$ and $b$ can be obtained by

$$P_{10} = m\omega^2 A_r, P_{11} = -6A_r, P_{13} = A_r,
$$

$$P_{12} = [-3A_r | A|^2 - 9B_1(A^2_r - A^2_s)] - 54A_r | B|^2 + 18A_r A_r B_1 | \omega|^3,$n

$$P_{14} = 3A_r | A|^2 + 6A_r | B|^2 + 3B_1(A^2_r - A^2_s) + 6B_1 A_r A_r,$n

$$P_{20} = m\omega^2 A_r, P_{21} = -6A_r,$n

$$P_{22} = [3A_r | A|^2 + 9B_1(A^2_r - A^2_s)] + 54A_r | B|^2 + 18A_r A_r B_1 | \omega|^3,$n

$$P_{23} = A_r P_{24} = A_r(A^2_r - A^2_s) - 2A_r A_r + 6B_1 | A|^2 + 3B_1 | B|^2,$n

$$P_{24} = 3A_r | A|^2 + 3A_r B_1 - 81B_1 | B|^2 - 18B_1 | A|^2 | \omega|^3,$n

$$P_{30} = B_r + P_{31} = A_r(A^2_r - A^2_s) - 2A_r A_r + 6B_1 | A|^2 + 3B_1 | B|^2,$n

$$P_{33} = 9m\omega^2 B_r, P_{32} = -3\omega B_1 P_{33} = (A^2_r + 3A_r A_r - 81B_1 | B|^2 - 18B_1 | A|^2) | \omega|^3,$n

$$P_{34} = B_r + P_{34} = A_r(A^2_r - A^2_s) + 2A_r A_r + 6B_1 | A|^2 + 3B_1 | B|^2.$$ (13)

The following special attentions should be paid in the parameters identification procedure

1. Parameter $P$ should be calculated under different excitation frequency, different excitation force amplitudes, different static load conditions. Firstly, record the response time series, calculate the Fourier coefficients, parameter $\omega$ can be obtained by (13); (2) To assure the accuracy of the system parameters identification, the data should be recorded near the resonance frequency, the data far from the resonance frequencies can be ignored.

(3) All the $T_f$ curves should be taken into account to obtain a general vibration transmissibility model.

Honeycomb paperboard specimen with different height ($H=20$mm, $H=30$mm, $H=40$mm, $H=50$mm) are tested and the parameters identification results are shown in Table 1.

**Table 1.** Identified dynamic property parameters of honeycomb paperboard under steady state response condition

<table>
<thead>
<tr>
<th>Specimen height(mm)</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$×10$^3$ (N/m)</td>
<td>9.116</td>
<td>8.077</td>
<td>7.184</td>
<td>6.152</td>
</tr>
<tr>
<td>$k$×10$^3$ (N/m$^2$)</td>
<td>-1.483</td>
<td>-4.521</td>
<td>-6.780</td>
<td>-7.510</td>
</tr>
<tr>
<td>$c$×10$^3$ (Ns/m)</td>
<td>1.375</td>
<td>1.284</td>
<td>1.673</td>
<td>1.573</td>
</tr>
<tr>
<td>$c$×10$^3$ (Ns/m$^2$)</td>
<td>6.538</td>
<td>7.365</td>
<td>9.749</td>
<td>9.205</td>
</tr>
</tbody>
</table>

The identified parameters in Table 1 can be used to simulate the response of the honeycomb paperboard-mass system under different static load and different excitation amplitude conditions, the $T_f$ curves can also be simulated by the identified parameters and the model presented by (2), which is important for anti-vibration packaging design. Substitute parameters in Table 1 into (2), one can obtain the response of the honeycomb paperboard-mass system $z$ by use of fourth order Runge-Kutta method[17], displacement response $x$ of the system can be obtained by

$$x(t) = z(t) + y(t) = z(t) + \ddot{y}(t)/\omega^2.$$

Then, the vibration transmissibility can be estimated by
\[ T_r = \frac{|\omega|}{|\omega|} \approx \frac{|\omega|}{|\omega|} \]

The simulated \( T_r-f \) curves under different static load and excitation force amplitude conditions are shown in Figure 4, the experimental transmissibility data are also shown in this figure, from this figure, the simulated transmissibility curves have good agreement with the experimental data. The simulation results are only for \( H=40 \text{mm} \), the \( T_r-f \) curves of different honeycomb paperboard heights can be obtained with the similar simulation procedures. The model for the vibration transmissibility of honeycomb paperboard and the identified parameters can be used to simulate \( T_r-f \) curves of honeycomb paperboard under different static load and different excitation force conditions, which is important for the proper use of honeycomb paperboard and anti-vibration packaging design.

**Figure 4.** Comparison of simulated transmissibility curves with experiment data
(a) \( m=5 \text{kg} \) (b) \( m=10 \text{kg} \) (c) \( m=15 \text{kg} \)

From the simulation result in Figure 4, the simulation is very accurate if the amplitude of excitation acceleration is small, while the excitation force is very big (for example, the amplitude of excitation acceleration is \( 39.2 \text{m/s}^2 \)), there exists about 5\% error between the simulation with the experimental data. The error indicates that the nonlinear elastic and damping model is not a perfect model for honeycomb paperboard, but this model is sufficient to simulate the vibration transmissibility property.

5. Conclusion

The dynamic property of honeycomb paperboard under vibration condition are different from that under the shock condition. Comparing with the identified results in [6,7], honeycomb paperboard has greater damping and stiffness properties under shock condition than those under vibration condition, and the properties under shock condition are more sensitive to the load than those under vibration condition. Therefore, it is necessary to model the dynamic properties of honeycomb paperboard under shock and vibration conditions respectively.

Honeycomb paperboard has different vibration properties under different excitation conditions. The experimental results indicate that as the excitation amplitude increases, the stiffness decreases and the damping increases, meanwhile, with the increase of excitation force amplitude, the resonance frequency shifts to the left and the resonance peak decreases. Therefore, we can assume both the damping and stiffness properties are nonlinear. Both the damping force and the elastic force are molded as the combination of the linear part with the cubic nonlinear part. A parameters identification process is formulated based on harmonic balance method, the parameters of the mass loaded honeycomb...
paperboard system under different load conditions are identified. The comparison of the simulated transmissibility curves with the experimental transmissibility indicate that the model is accurate, it can be used to simulate the $T_f$ curves of honeycomb paperboard under different load conditions.

6. Acknowledgement

This work is supported by the National Natural Science Foundation of China under the Grant 61064012, Gansu Provincial Natural Science Foundation under the Grant 1014RJZA047, basic Scientific Research Special Fund of Gansu Institution of Higher Education.

7. References