Models for Multiple Attribute Decision Making with Intuitionistic Trapezoidal Fuzzy Information

Guorong Xiao
Department of Computer Science and Technology, GuangDong University of Finance, Guangzhou, 510521, China, E-mail: newducky@126.com

Abstract

The aim of this paper is to investigate the multiple attribute decision making problems to deal with supplier selection with intuitionistic trapezoidal fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy number. We developed a multiple attribute decision making method by closeness to ideal alternative in intuitionistic trapezoidal fuzzy setting. Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives. Finally, an example with supplier selection is given.

Keywords: Multiple Attribute Decision Making, Supplier Selection, Intuitionistic Trapezoidal Fuzzy Number, Distances Measure

1. Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [2] whose basic component is only a membership function. The intuitionistic fuzzy set has received more and more attention since its appearance [3-10]. Later, Atanassov and Gargov [11-12] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Xu [13-14] developed some aggregation operators with interval-valued intuitionistic fuzzy information. Wang [15] investigated the interval-valued intuitionistic fuzzy MADM with incompletely known weight information. A nonlinear programming model is developed. Then, using particle swarm optimization algorithms to solve the nonlinear programming models, the optimal weights are gained. And ranking is performed through the comparison of the distances between the alternatives and ideal/anti-ideal alternative. Shu, Cheng and Chang [16] gave the definition and operational laws of intuitionistic triangular fuzzy number and proposed an algorithm of the intuitionistic fuzzy fault-tree analysis. Wang [24] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic trapezoidal fuzzy number. Wang and Zhang [17] gave the definition of expected values of intuitionistic trapezoidal fuzzy number and proposed the programming method of multi-criteria decision-making based on intuitionistic trapezoidal fuzzy number. And the Hamming distance of intuitionistic trapezoidal fuzzy numbers and intuitionistic trapezoidal fuzzy weighted averaging (ITFWAA) operator, then proposed multi-criteria decision-making method with incomplete certain information based on intuitionistic trapezoidal fuzzy number.

The aim of this paper is to investigate the multiple attribute decision making problems to deal with supplier selection with intuitionistic trapezoidal fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy number. We developed a multiple attribute decision making method by closeness to ideal alternative in intuitionistic trapezoidal fuzzy setting. Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives. Finally, an example with supplier selection is given.
alternatives. In Section 4, a practical example about supplier selection is provided to illustrate the proposed method. In Section 5 we conclude the paper and give some remarks.

2. Preliminaries

In the following, we shall introduce some basic concepts related to intuitionistic trapezoidal fuzzy numbers.

**Definition 1.** Let $\tilde{a}$ be an intuitionistic trapezoidal fuzzy number, its membership function is [15-17]:

$$
\mu_{\tilde{a}}(x) = \begin{cases} 
\frac{x-a}{b-a} \mu_b, & a \leq x < b; \\
\frac{b-x}{b-a} \mu_{\tilde{a}}, & b \leq x \leq c; \\
\frac{d-x}{d-c} \mu_{\tilde{a}}, & c < x \leq d; \\
0, & \text{others.}
\end{cases}
$$

(1)

its non-membership function is:

$$
\nu_{\tilde{a}}(x) = \begin{cases} 
\frac{b-x+\nu_{\tilde{a}}(x-a)}{b-a} \nu_{\tilde{a}}, & a \leq x < b; \\
\frac{c-x}{c-d} \nu_{\tilde{a}}, & c < x \leq d; \\
0, & \text{others.}
\end{cases}
$$

(2)

where $0 \leq \mu_{\tilde{a}} \leq 1, 0 \leq \nu_{\tilde{a}} \leq 1$ and $\mu_{\tilde{a}} + \nu_{\tilde{a}} \leq 1; a, b, c, d \in \mathbb{R}$.

Then $\tilde{a} = \left( \left[ a, b, c, d \right]; \mu_{\tilde{a}}, \nu_{\tilde{a}} \right)$ is called an intuitionistic trapezoidal fuzzy number.

For convenience, let $\tilde{a} = \left( \left[ a, b, c, d \right]; \mu_{\tilde{a}}, \nu_{\tilde{a}} \right)$.

**Definition 2.** Let $\tilde{a}_1 = \left( \left[ a_1, b_1, c_1, d_1 \right]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1} \right)$ and $\tilde{a}_2 = \left( \left[ a_2, b_2, c_2, d_2 \right]; \mu_{\tilde{a}_2}, \nu_{\tilde{a}_2} \right)$ be two intuitionistic trapezoidal fuzzy numbers, and $\lambda \geq 0$, then [15-17]

1. $\tilde{a}_1 + \tilde{a}_2 = \left( \left[ a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 \right]; \mu_{a_1} + \mu_{a_2}, \nu_{a_1} + \nu_{a_2} \right)$;
2. $\tilde{a}_1 \cdot \tilde{a}_2 = \left( \left[ a_1 \cdot a_2, b_1 \cdot b_2, c_1 \cdot c_2, d_1 \cdot d_2 \right]; \mu_{a_1} \cdot \mu_{a_2}, \nu_{a_1} \cdot \nu_{a_2} \right)$;
3. $\lambda \tilde{a}_1 = \left( \left[ \lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1 \right]; 1 - (1 - \mu_{a_1})^\lambda, \nu_{a_1}^\lambda \right)$;
4. $\tilde{a}_1^\lambda = \left( \left[ a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda \right]; \mu_{a_1}^\lambda, 1 - (1 - \nu_{a_1})^\lambda \right)$

**Definition 3.** Intuitionistic trapezoidal fuzzy positive ideal solution and intuitionistic trapezoidal fuzzy negative ideal solution are defined as follows:

$$\bar{r}^+ = \left( \left[ a^+, b^+, c^+, d^+ \right]; \mu^+, \nu^+ \right) = \left( \left[ 1, 1, 1, 1 \right]; 1, 0 \right)$$

(3)
Definition 4. Let \( \tilde{a}_1 = ([a_1, b_1, c_1, d_1]; \mu_{a_1}, \nu_{a_1}) \) and \( \tilde{a}_2 = ([a_2, b_2, c_2, d_2]; \mu_{a_2}, \nu_{a_2}) \) be two intuitionistic trapezoidal fuzzy number, then the normalized Hamming distance between \( \tilde{a}_1 \) and \( \tilde{a}_2 \) is defined as follows[17]:

\[
\begin{align*}
    d(\tilde{a}_1, \tilde{a}_2) &= \frac{1}{8} \left[ \left| (1 + \mu_{a_1} - \nu_{a_1}) a_1 - (1 + \mu_{a_2} - \nu_{a_2}) a_2 \right| + \left| (1 + \mu_{a_1} - \nu_{a_1}) b_1 - (1 + \mu_{a_2} - \nu_{a_2}) b_2 \right| \\
    &\quad + \left| (1 + \mu_{a_1} - \nu_{a_1}) c_1 - (1 + \mu_{a_2} - \nu_{a_2}) c_2 \right| + \left| (1 + \mu_{a_1} - \nu_{a_1}) d_1 - (1 + \mu_{a_2} - \nu_{a_2}) d_2 \right| \right].
\end{align*}
\]

(4)

3. Models for multiple attribute decision making with intuitionistic trapezoidal fuzzy information

Let \( A = \{A_1, A_2, \cdots, A_m\} \) be a discrete set of alternatives, and \( G = \{G_1, G_2, \cdots, G_n\} \) be the set of atributes. Suppose that \( \bar{R} = (\tilde{r}_{ij})_{m \times n} = ([a_{ij}, b_{ij}, c_{ij}, d_{ij}; \mu_{ij}, \nu_{ij}]_{m \times n} \) is the intuitionistic trapezoidal fuzzy decision matrix, \( \mu_{ij}^{(k)} \in [0,1], \nu_{ij}^{(k)} \in [0,1], \mu_{ij}^{(k)} + \nu_{ij}^{(k)} \leq 1, \ i = 1,2,\cdots,m, \ j = 1,2,\cdots,n, \ k = 1,2,\cdots,t \). The information about attribute weights is incompletely known. Let \( w = (w_1, w_2, \cdots, w_n) \) be the weight vector of attributes, where \( w_j \geq 0, \ j = 1,2,\cdots,n \), \( \sum_{j=1}^{n} w_j = 1 \).

Step 1. Suppose that \( \bar{R} = (\tilde{r}_{ij})_{m \times n} = ([a_{ij}, b_{ij}, c_{ij}, d_{ij}; \mu_{ij}, \nu_{ij}]_{m \times n} \) is the intuitionistic trapezoidal fuzzy decision matrix, \( \mu_{ij}^{(k)} \in [0,1], \nu_{ij}^{(k)} \in [0,1], \mu_{ij}^{(k)} + \nu_{ij}^{(k)} \leq 1, \ i = 1,2,\cdots,m, \ j = 1,2,\cdots,n, \ k = 1,2,\cdots,t \). The information about attribute weights is incompletely known. Let \( w = (w_1, w_2, \cdots, w_n) \) be the weight vector of attributes, where \( w_j \geq 0, \ j = 1,2,\cdots,n \), \( \sum_{j=1}^{n} w_j = 1 \).

Step 2. Let \( \bar{R} = (\tilde{r}_{ij})_{m \times n} = ([a_{ij}, b_{ij}, c_{ij}, d_{ij}; \mu_{ij}, \nu_{ij}]_{m \times n} \) be the intuitionistic trapezoidal fuzzy decision matrix, the ideal alternative can be defined as follows:

\[
A^+ = (\tilde{r}_{ij}^+, \tilde{r}_{ij}^+, \cdots, \tilde{r}_{ij}^+), \text{ where } \tilde{r}_{ij}^+ = ([a_{ij}, b_{ij}, c_{ij}, d_{ij}; \mu_{ij}^+, \nu_{ij}^+] = ([1,1,1,1];1,0).
\]

Step 3. Utilize the weight vector \( w = (w_1, w_2, \cdots, w_n) \) and utilize the (4) to derive the distances \( d(A_i, A^+)(i = 1,2,\cdots,m) \), by which we can get the ranking of all alternatives \( A(i = 1,2,\cdots,m) \).

\[
d(A_i, A^+) = \sum_{j=1}^{n} w_j d(\tilde{r}_{ij}^+, \tilde{r}_{ij}^+)
\]

(5)
Step 4. Rank all the alternatives $A_i (i = 1, 2, \cdots, m)$ and select the best one(s) in accordance with $d(A_i, A^+)(i = 1, 2, \cdots, m)$: the smaller the better $d(A_i, A^+)$, the better the alternatives $A_i$.

Step 5. End.

4. Illustrative Example

Let us suppose there is a problem to deal with the supplier selection in supply chain management. There are five prospect suppliers $A_i (i = 1, 2, 3, 4, 5)$ for four attributes $G_j (j = 1, 2, 3, 4)$. The four attributes include product quality $G_1$, service $G_2$, delivery $G_3$, and price $G_4$, respectively. The five possible suppliers alternatives $A_i (i = 1, 2, 3, 4, 5)$ are to be evaluated using the intuitionistic trapezoidal fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$
\tilde{R} = \begin{bmatrix}
(0.5, 0.6, 0.8, 0.9; 0.3, 0.6) & (0.1, 0.2, 0.3, 0.4; 0.6, 0.3) \\
(0.4, 0.5, 0.7, 0.8; 0.7, 0.2) & (0.5, 0.6, 0.7, 0.8; 0.7, 0.2) \\
(0.5, 0.6, 0.7, 0.8; 0.5, 0.3) & (0.2, 0.3, 0.5, 0.6; 0.5, 0.4) \\
(0.1, 0.3, 0.5, 0.7; 0.3, 0.4) & (0.1, 0.3, 0.4, 0.5; 0.6, 0.3) \\
(0.2, 0.3, 0.4, 0.5; 0.7, 0.1) & (0.3, 0.4, 0.5, 0.6; 0.4, 0.3) \\
(0.5, 0.6, 0.7, 0.8; 0.5, 0.4) & (0.4, 0.5, 0.6, 0.7; 0.2, 0.7) \\
(0.6, 0.7, 0.8, 0.9; 0.7, 0.3) & (0.5, 0.6, 0.7, 0.9; 0.4, 0.5) \\
(0.1, 0.2, 0.4, 0.5; 0.6, 0.4) & (0.3, 0.5, 0.7, 0.9; 0.2, 0.3) \\
(0.3, 0.4, 0.5, 0.6; 0.8, 0.1) & (0.6, 0.7, 0.8, 0.9; 0.2, 0.6) \\
(0.2, 0.3, 0.4, 0.5; 0.6, 0.2) & (0.5, 0.6, 0.7, 0.8; 0.1, 0.3)
\end{bmatrix}
$$

Then, we utilize the approach developed to get the most desirable alternative(s).

Step 1. If the information about the attribute weights is known and the known weight information is given as follows:

$$
w = (0.1, 0.3, 0.2, 0.4)
$$

Step 2. Utilize the weight vector $w$ and utilize the (4) to derive the distances $d(A_i, A^+)(i = 1, 2, 3, 4, 5)$, by which we can get the ranking of all alternatives $A_i (i = 1, 2, 3, 4, 5)$:

$$
d(A_1, A^+) = 0.7920, d(A_2, A^+) = 0.5714, d(A_3, A^+) = 0.7570
$$

$$
d(A_4, A^+) = 0.7105, d(A_5, A^+) = 0.7193
$$

Step 3. Rank all the alternatives $A_i (i = 1, 2, 3, 4, 5)$ in accordance with the distances $d(A_i, A^+)$: $A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5$, and thus the most desirable alternative is $A_i$.

5. Conclusion

In this paper, we have in investigated the multiple attribute decision making problems to deal with...
supplier's election with intuitionistic trapezoidal fuzzy information, in which the information about the attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy number. We developed a multiple attribute decision making method by closeness to ideal alternative in intuitionistic trapezoidal fuzzy setting. Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives. At last, a practical example about supplier selection is provided to illustrate the proposed method.

6. References