**Image Denoising based on Adaptive BM3D and Singular Value Decomposition**

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**Abstract**

In this work a new version of block-matching and 3D filtering (BM3D) denoising approach introduced by Dabov et al. for denoising the image corrupted by additive white Gaussian noise is proposed. The BM3D performs collaborative filtering to the 3D image groups composed by similar image blocks with the fixed hard-thresholding operator. The proposed version of BM3D adopts adaptive block-matching threshold in the block-matching step and the denoising method based on singular value decomposition is used before applying BM3D as the performance of BM3D falls rapidly to strong noise image. To sum up, the proposed method firstly exploits the noise estimation to get the noise level of the given image. Then singular value decomposition is applied to pre-filtering to the high noise level image. Finally BM3D denoising method algorithm with adaptive block-matching thresholds is adopted. Experiment results are given to show that the proposed algorithm achieves better denoising performance than the original BM3D.

**Keywords**: Non-Local Means, Singular Value, BM3D, Structural Similarity Index

1. Introduction

Image denoising is one of the classical Image Processing technologies, many researchers had studied it for a long time [1-3]. Recent years, image denoising has been impacted by Non Local Means (NLM) [4] which relies on the use of overcomplete dictionaries which learned from the noisy image or from a larger data set [5]. The state-of-the-art denoising algorithm BM3D based on NLM offers remarkably promising results. The process of BM3D includes two main steps: grouping and collaborative filtering. Similar matched 2D blocks are found mutually and stacked together to form a 3D array; the collaborative filtering takes advantage of the increased correlation in the formed 3D array to effectively suppress the noise and produce estimates of the true signal from the grouped blocks [6]. As the BM3D applies the fixed block-matching threshold, the strong threshold is time-consuming for mildly noisy image while the weak threshold performs an unsatisfied result for seriously noisy image. Moreover, the denoising performance has a sharp drop when noise standard deviation reaches 40 [7]. The authors of [8] develop other algorithms to improve performance by using DCT transform instead of wavelet or changing many coefficients which lead to discontinuity in mathematics and bring artificial impact to the result. The proposed technique automatically applies weak thresholds for mildly noisy images while strong thresholds together with SVD pre-filtering are adopted for high noise level images.

2. Singular Value Decomposition (SVD)

In linear algebra, Singular Value Decomposition (SVD) is a factorization of a real or complex matrix, with many useful applications in signal processing and statistics. Using SVD, a complex matrix M is written as:

\[ M = U \Sigma V^* \]

and this can be expressed in alternative form as:

\[ M = \sum_{i=1}^{N} s_i u_i v_i^* \]
where \( u_i \)'s are the columns of the output basis, \( v_i \)'s are the rows of the input basis, and \( s_i \)'s are called the singular values which form the diagonal of the \( M \) matrix. The singular values can be thought of as a weighting factor of the component images if \( M \) is an image [9].

As SVD is the statistical representation of \( M \) in subspaces of decreasing importance, only using the first dozens of singular values can be reconstructed a closest matrix to \( M \).

### 3. Hard Block-matching Threshold in BM3D

Given a noisy image \( z : z(x) = y(x) + \eta(x) \), where \( x \) is a 2D spatial coordinate that belongs to image domain, \( y \) is the true image, and \( \eta \) is i.i.d. zero-mean Gaussian noise with variance \( \sigma^2 \), \( \eta(x) \sim N(0, \sigma^2) \). \( Z_x \) is denoted as a block of fixed size extracted from \( z \), and \( d(Z_{xg}, Z_x) \) is defined as the block-matching distance of the reference block \( Z_{xg} \) with size \( N \times N \):

\[
d(Z_{xg}, Z_x) = \frac{\| \Psi'(T_{2D}(Z_{xg}) - \Psi'(T_{2D}(Z_x)) \|_2}{N},
\]

where \( T_{2D} \) is a normalized 2D linear transform operator such as DCT, \( \Psi' \) is a hard-threshold operator and \( \| \cdot \|_2 \) denotes the \( \ell^2 \)-norm [8]. A set \( S_{xg} \) whose blocks are similar to \( Z_{xg} \) is obtained:

\[
S_{xg}^{ht} = \{ (z, x) \in S_{xg} : d(Z_{xg}, Z_x) \leq \tau_{match} \}.
\]

In the paper [8], the parameter \( \tau_{match} \) is determined completely with experience analysis, ignoring the different noisy components of the signal. Overlarge threshold results in the meaningless increasing of the time complexity and computational complexity as information is overcomplete for low noise level image, while too small block-matching distance leads to reducing the numbers of matched blocks which falls to 2D domain when no block is matched [10]. It is feasible to limit the number of matched blocks to improve the performance of time-consuming when the added noise level is low; however, when noise standard deviation is higher than 40, the author of [8] try to adopt many different parameters to improve the denoising performance. We propose an adaptive threshold algorithm with just a simple threshold computational formula to reach a satisfactory performance without modifying other parameters.

### 4. Adaptive Block-matching Threshold

By using the algorithm of the Structural Similarity index (SSIM) [11], we put up a simple relationship of two common noisy image features: gradient values and the estimated noise level. SSIM is a method for measuring the similarity between two images which outperforms the traditional methods like peak signal-to-noise ratio (PSNR) and mean squared error (MSE).

For the reference block \( Z_{xg} \) of the noise free image, we firstly build a relationship between the block-matching distance \( d(Z_{xg}, Z_x) \) and the SSIM value of the candidate block \( Z_x \). Then fine SSIM values are selected from the data obtained in the first step which means two selected blocks are preferably similar. After that, the noise free image is added with zero-mean Gaussian noise, and the noise estimation together with the computation of gradient value is adopted to the same blocks accordingly. Finally, apply surface fitting to get the simple formula whose threshold varies with different noisy images.

Given a noisy image with block size of \( N \times N \), the Structural Similarity index (SSIM) is defined as:
\[
SSIM(Z_{xg}, Z_x) = \frac{(2\mu_{xg}\mu_{x} + C_1)(\sigma_{xg}\sigma_x + C_2)}{\left(\mu_{xg}^2 + \mu_{x}^2 + C_1\right)\left(\sigma_{xg}^2 + \sigma_x^2 + C_2\right)}.
\] (4)

where \(\mu_{xg}\), \(\mu_x\) is the mean intensity of \(Z_{xg}\) and \(Z_x\) respectively, \(\sigma_{xg}\), \(\sigma_x\) is the standard deviation, \(\sigma_{xg}\sigma_x\) is the correlation coefficient corresponds to the cosine of the angle between the vectors \(Z_{xg} - \mu_{xg}\) and \(Z_x - \mu_x\). \(C_1\) and \(C_2\) are small constants which eliminates unstable results when either \(\mu_{xg}^2 + \mu_x^2\) or \(\sigma_{xg}^2 + \sigma_x^2\) is very close to zero [11]. \(V_{SSIM}\), \(D_{BM3D}\) is denoted as the value of SSIM and gradient respectively, then we get the following relationship as:

\[V_{SSIM} = f(D_{BM3D}).\] (5)

Figure 1. Relationship between block-matching distance and SSIM

In general, the relationship between SSIM value and block-matching distance would probably present reciprocal as SSIM varies according to the similarity of compared blocks (images) and approaches to its maximum value of 1 when two blocks being compared are exactly the same while Euclidean distance decreases when two blocks are more similar. Figure 1 shows that for the noise free image, SSIM keeps at the high level which falls rapidly along with the increase of block-matching distance.

We denote \(D_{SSIM}\) as the block-matching distance with preferable SSIM values and then add variable noise to the noise free image to create different noisy images. Noise estimation and gradient value computation to each block are adopted to get the estimation of added noise \(\sigma_{estimate}\) and its gradient value \(g\). Then we build the threshold formula as follows:

\[D_{SSIM}(n) = f(g(n), \sigma_{estimate}(n)),\] (6)

where \(g(n)\), \(\sigma_{estimate}(n)\) is the gradient value and estimated noise level of \(n\)-th block respectively. After adopting surface fitting to the data of 100 standard images, the above formula is written as:
\[ D_{\text{SSIM}}(n) = 12.15 + 1.464 \sigma_{\text{estimate}} - 0.4383 \gamma(n). \]  

(7)

And for the whole image, the formula can be written as:

\[ D_{\text{SSIM}} = 12.15 + 1.464 \sigma_{\text{estimate}} - 0.4383 \gamma, \]

(8)

where \( \sigma_{\text{estimate}} \) and \( \gamma \) are the mean value of the image's noise level and gradient respectively.

5. Experiments and Discussions

We test denoising algorithms on various noisy images corrupted by Gaussian noise with standard deviation \( \sigma \) ranging from 10 to 100. The adaptive BM3D and adaptive BM3D based on Singular Value Decomposition are denoted as A-BM3D and SVD-BM3D respectively. In order to make the algorithm more consistent and avoid introducing much periodic artifacts, the parameter setup for strong noisy image should be used just the same as the situation of the weak noisy image [7]. Therefore we use the fixed parameters defined in [8], while replace the parameter \( \tau_{\text{match}} \) with the formula (8) in A-BM3D and SVD-BM3D. Table 1 and Fig. 2 present the PSNR performance of above-mentioned algorithms, and Fig. 3 gives the relative PSNR improvement of three algorithms.

**Table 1.** PSNR (dB) performance with noise ranging from 10 to 100

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>BM3D</th>
<th>A-BM3D</th>
<th>SVD-BM3D</th>
<th>BM3D</th>
<th>A-BM3D</th>
<th>SVD-BM3D</th>
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<td>40</td>
<td>27.194</td>
<td>27.108</td>
<td>27.108</td>
<td>29.933</td>
<td>29.924</td>
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<tr>
<td>70</td>
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<td>24.330</td>
<td>24.386</td>
<td>25.803</td>
<td>27.262</td>
<td>27.329</td>
</tr>
<tr>
<td>100</td>
<td>21.582</td>
<td>22.609</td>
<td>22.801</td>
<td>23.761</td>
<td>25.433</td>
<td>25.623</td>
</tr>
</tbody>
</table>

**Figure 2.** Denoising performance on PSNR (dB)
Table 1 and Fig. 2 show that in the presence of weak noise, BM3D works well and stable which becomes unreliable with the increase of additive noise as the BM3D is not able to reach a sufficiently sparse representation. Moreover, the performance of BM3D falls rapidly when the noise standard deviation exceeds 40 which the matched blocks are not really similar in the noise free image and this situation is described as erroneous grouping by the authors of [1]. The proposed algorithms perform more stably and satisfactory than BM3D especially for the strong noisy image as the A-BM3D and SVD-BM3D adjust their block-matching thresholds according to different noise levels or images. What’s more, especially to the strong noise images, SVD-BM3D works better than A-BM3D by applying SVD pre-filtering to noisy image which can be seen from Fig. 3.

It is known that SVD reconstructs the signal by using only a part of information (singular values), and it will lead to some loss to the quality of the results. Although the SVD pre-filtering works well for strong noise, it is not recommended to apply SVD pre-filtering for weak noise as denoising results are already satisfactory and introducing SVD may reduce the quality of the image. What’s more, the selection of the numbers of singular values also affects the denoising performance which too many or too few numbers singular values selection both lower the SVD denoising result. Because too many numbers of singular values lead to the overmuch preservation of noise after reconstruction, while too few numbers result in the excessive loss of image quality. In our paper, after many experiments, we adopted a preferable number with about 0.586 ratios to the pixel of given image. That is to say, for an image with the size of 512*512, we adopt about 300 singular values to reconstruct the image.

And one should be mentioned is that time complexity of proposed algorithms is lower than BM3D although the denoising performance is slightly bad. In the present of weak noisy image, the block-matching threshold adopted by original BM3D algorithm is overlarge as the information is overcomplete or spare enough. Overlarge threshold expands unnecessary time on searching the similar blocks locating far from reference block. However, with the raise of noise level, the fixed threshold becomes too small which leads to the fall of denoising performance.
Fig. 4 shows that when the image is seriously noised (\( \sigma = 70 \)), SVD-BM3D exceeds original BM3D both in objective image quality standardization and subjective visual effect. It can be seen that, comparing with BM3D, the result of SVD-BM3D works satisfactorily in texture region and smoothing region as texture region is more clearly demarcated and smooth region is much smoother. What’s more, the result of BM3D is contaminated with corrugated stripes.

In all, by adjusting block-matching threshold automatically, SVD-BM3D performs more stably and satisfactorily than original BM3D which consumes less time to weak noisy image. And for strong noisy image, thanks to the SVD pre-filtering, SVD-BM3D achieves more excellent than the original BM3D with more clearly demarcated texture region and smoother smooth region.

6. Conclusions

The proposed SVD-BM3D represents more outstanding and robust than original BM3D especially to the strong noisy images. The use of fixed parameters (except for block-matching threshold) and SVD pre-filtering instead of changing the filtering in block-matching step avoids the discontinuity in algorithm and mathematics which may lead to the wild fluctuation to the denoising performance.

References
[9] Information on http://www.u.arizona.edu/~ppoon/