Uniform Attractors for Non-Autonomous Strongly Damped Equations

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Abstract
In this paper, the vibration of beam equations with strongly damped term when the driving force is time-dependent is discussed. Based on the semigroup theorem and the technique of decomposing solutions, we prove the existence of compact kernel sections for the solution process. And under the homogeneous boundary condition, we obtain the existence of uniform attractors in a certain two-parameter region.

Keywords: Non-autonomous; strongly damped; Beam equation; Uniform attractors; Absorbing set

1. Introduction

It is well known that the investigation of the asymptotic behavior of solutions for nonlinear hyperbolic equations is one of the main problems in the non-autonomous infinite dimensional dynamical systems. Recently, the study of the uniform attractor for non-autonomous wave equation with strongly damped term has attracted much attention of many mathematicians and has made fast progress. Wave equations, describing a great variety of wave vibration phenomena, occur in the applications of natural phenomenon. And, there are many works on attractors in the literature [1-2].

For autonomous case, Andrew Comech studied the Klein-Gordon under certain generic assumptions and proved the convergence to the attractor [3]. The global attractor for the wave equation with displacement dependent damping was studied, if the damping coefficient function is strictly positive near the origin [4, 5]. Under the semilinear porous acoustic boundary conditions, the model of the wave equation with nonlinear boundary/interior sources and damping was studied by Philip Jameson Graber and Belkacem Said-Houari [6-10].

On the other hand, when the case is non-autonomous system, the existence of compact kernel sections for the process generated by a non-autonomous strongly damped wave equation with homogeneous Dirichlet boundary condition was proved [11-15]. And, the upper bound of the Hausdorff dimension of sections decreases as the damping grows for large strong damping was discussed [10]. When the equation is modeled by non-autonomous beam system, Li Zhiyu and Ma Qiaozhen proved the existence of uniform attractors for the more general non-autonomous extensible beam equations in weak and strong topology spaces [16-19]. In [17, 20-22], a new concept condition (C*) was introduced, and the set of all functions satisfying condition (C*) was denoted. In the paper, the authors showed that there are many functions satisfying condition (C*); meanwhile, they gave an application, the existence of uniform attractor for non-autonomous wave equations involving mixed differential quotient terms was obtained. Under some restrictions on $\beta(t), \gamma(t)$ and growth restrictions on the nonlinear term $f$, Tomas Caraballo, Alexandre N. Carvalho, Jose A. Langa, Felipe Rivero consider the strongly damped wave equation with time-dependent terms in a bounded domain, they obtained the existence and regularity of pullback attractors(see[23-25]).

The aim of this paper is to discuss the existence of uniform attractors for the non-autonomous system in $E$ under certain conditions. By introducing a new two-parameter operator families, we study the following non-autonomous strongly damped beam equation with a time-dependent external force in an sufficiently smooth bounded open region $\Omega \subset R^1$ with boundary $\partial \Omega$, arising in the model of a generalized beam:

$$u_{tt} + \Delta^2 u + \gamma \Delta u + g(u) + h(u) = g(x,t), \quad x \in \Omega, \quad t \geq \tau, \quad \tau \in R,$$

Associated with the following initial value conditions:
\( u(x, \tau) = u_{\tau}(x) \in H_{0}^{2}(\Omega), \quad x \in \Omega, \)  
(2)

And homogeneous Dirichlet boundary conditions:

\( u(x, t) = \Delta u(x, t) = 0, \quad x \in \partial \Omega, \quad t \geq \tau, \quad \tau \in \mathbb{R}, \)  
(3)

Where the unknown function \( u(x, t) \) depends on variables \( x \) and \( t \) on \( \Omega \times [\tau, +\infty) \), and Equation (1) represents the perturbed equation. \( \gamma \) is a non-negative constant. The time-dependent function \( q(x, t) \) represents the external forcing term. Then, we give some hypotheses on the non-autonomous term \( q(x, t) \) and the nonlinear terms \( g(u), h(u) \) satisfy the following conditions:

(H1) \( q(x, t) \in C_{b}(R, L^{2}(\Omega)), \)

(H2) \( h \in C^{1}(R, R), \)

and

\[ |h(u)| \leq c_{1}, \quad |h(u_{1}) - h(u_{2})| \leq c_{2} |u_{1} - u_{2}|, \quad \forall u_{1}, u_{2} \in R, \]  
(4)

where \( c_{1}, c_{2} \geq 0 \) are two positive constants

(H3) \( g \in C^{1}(R, R), \quad g(0) = 0, \)

and

\[ -\gamma \lambda_{i} < \beta_{i} \leq g'(s) \leq \beta_{i} < +\infty, \quad \forall s \in R, \]  
(5)

where \( \beta_{i}, \lambda_{i} \geq 0 \) are two constants and \( \lambda_{i} \) is the first eigenvalue of the operator \( -\Delta \) on \( \Omega \).

2. Existence of uniform attractors

In this section, we obtain the existence, uniqueness of the system (1)-(3), and show the existence of a uniform attractor [26, 27].

To better explain the existence of uniform attractors for the non-autonomous infinite dimensional dynamical equations we need to recall some basic results. Let \( X \) be a metric space, and \( u(t, \tau; u_{\tau}) \) be an element in the phase space. It is said that \( \{u(t, \tau; u_{\tau})\} \) be a ‘‘non-autonomous dynamical system’’ if for any \( u_{\tau} \in X \)

\[ u(t, \tau; u_{\tau}) = u_{\tau} ; \]

\[ u(t, s; u(s, \tau; u_{\tau})) = u(t + s, \tau; u_{\tau}) . \]

An non-autonomous evolution process \( \{U_{\sigma}(t, \tau) : t \geq \tau\}, \sigma \in \Sigma \) in \( X \) is a family of mapping from \( X \) into itself with the following properties

(1) \( U_{\sigma}(t, \tau) = I_{\sigma} \) is the identity operator, for all \( \tau \in \mathbb{R} \);
(2) \( U_{\sigma}(t, \tau) = U_{\sigma}(t, s) \cdot U_{\sigma}(s, \tau), \) for all \( t \geq s \geq \tau, \tau \in \mathbb{R} \);
(3) \( (t, \tau, x) \in \{t, \tau \in \mathbb{R}^{+} : t \geq \tau\} \times \Omega \) \( \Rightarrow U_{\sigma}(t, \tau)x \in X \) is continuous.

**Definition 1** (see [17]) A closed set \( A_{\Sigma} \subset X \) is said to be the uniform(w.r.t. \( \sigma \in \Sigma \))attractor of the family of process \( \{U_{\sigma}(t, \tau) : t \geq \tau\}, \sigma \in \Sigma \) if it is uniformly(w.r.t. \( \sigma \in \Sigma \))attracting (attracting property) and it is contained in any closed uniformly(w.r.t. \( \sigma \in \Sigma \))attractor \( \mathcal{A}' \) of the family of processes \( \{U_{\sigma}(t, \tau) : t \geq \tau\}, \sigma \in \Sigma \) : \( A_{\Sigma} \subset \mathcal{A}' \) (minimality property).

Through the paper, we can let the linear operator \( A = \Delta^{2} \) is a self-adjoint, positive operator with the region

\[ \mathcal{D}(A) = H_{0}^{2}(\Omega) \cap H^{4}(\Omega). \]

And, let the Hilbert spaces:

\[ H = L^{2}(\Omega), \quad V = H_{0}^{2}(\Omega), \quad E = V \times H, \]

and endow these spaces with the usual scalar products and norms, respectively:
(u, v) = \int_{\Omega} u(x) \cdot v(x) dx, \quad \|u\|^2 = (u, u), \quad \forall u, v \in L^2(\Omega);
\|(u, v)\| = \int_{\Omega} \Delta u(x) \cdot \Delta v(x) dx, \quad \|v\|^2 = ((u, v)), \quad \forall u, v \in H^2_0(\Omega);
and
\[ (Z_1, Z_2)_E = ((u_1, u_2) + (v_1, v_2)), \quad \|Z_i\|^2 = \|u_i\|^2 + \|v_i\|^2, \quad \forall Z_i = \begin{pmatrix} u_i \\ v_i \end{pmatrix}, \quad Z = \begin{pmatrix} u \\ v \end{pmatrix} \in E, \quad i = 1, 2; \]

Let \( v = u \), it is convenient to express system (1)-(3) as an abstract first-order ordinary differential equation with respect to time \( t \) in the Hilbert space \( E \):
\[
\begin{align*}
\dot{Z} &= C_Y + F(Z, t), \quad x \in \Omega, \quad t \geq \tau, \quad \tau \in \mathbb{R}, \\
Z(\tau) &= Z_\tau = (u_{\omega}, u_0)^T \in E,
\end{align*}
\]
where
\[
\begin{align*}
Z &= \begin{pmatrix} u \\ v \end{pmatrix} \in E, \\
F(Z, t) &= \begin{pmatrix} 0 \\ -g(u) - h(u) + q(x, t) \end{pmatrix}, \\
C &= \begin{pmatrix} 0 & I \\ -\Delta^2 & -\gamma \Delta^2 \end{pmatrix}, \\
D(C) &= \left\{ (u, v)^T : u, v \in H^2_0(\Omega), u + \gamma v \in D(A) \right\}.
\end{align*}
\]

By [15], it is easy to know that \( C \) is a sectorial operator on \( E \) and \( e^{\rho t} \) is an analytic semigroup on \( E \). Meanwhile, we can check that \( F(Z, t) : E \to E \) is globally Lipschitz continuous with respect to \( Z \). Thus, we have following result:

**Lemma 1.** Assume that the conditions (H1)-(H3) hold. Then for \( \forall Z_\tau \in E \), there exists a unique function \( Z(t) = Z(t, Z_\tau) \in C([\tau, +\infty); E) \) such that \( Z_\tau = Z(\tau, Z_\tau) \) and \( Z(t) \) satisfies the following integral equation:
\[
Z(t, Z_\tau) = e^{(t-\tau)C} Z_\tau + \int_{\tau}^{t} e^{(t-s)C} F(Z(s), s) ds.
\]
\[
(7)
\]
\( Z(t, Z_\tau) \) is jointly continuous in \( t \) and \( Z_\tau \) and \( \forall T_\tau > 0, \)
\[
\begin{align*}
(u, v) &\in C([\tau, +\infty); H^2_0(\Omega)) \\
&\times \left[ C([\tau, +\infty); L^2(\Omega)) \cap L^2((\tau, +\infty); H^2_0(\Omega)) \right].
\end{align*}
\]

From Lemma 1, we know that \( Z(t) \) can be represented as
\[
Z(t) = U(t, \tau) Z_\tau, \\
Z_\tau \in E, \quad t \geq \tau, \quad \tau \in \mathbb{R},
\]

Where the mapping \( \{ U(t, \tau), t \geq \tau \} \) acting on \( E \) is a operator families with two parameters. And, it is said to be the process corresponding to system (1)-(3), and satisfies:
(1) \( U(t, \tau) = I \) is the identity on \( E \);
(2) \( U(t, \tau) = U(t, s) \cdot U(s, \tau), \quad t \geq s \geq \tau. \)

To construct the uniform attractor for system (1)-(3), we can set \( w = u_{\epsilon} + \epsilon u, \varphi = (u, w)^T \), where \( \epsilon \) is chosen as:
\[
\epsilon = \frac{\gamma \lambda^2 + \beta_1}{4 + 2(\gamma \lambda^2 + \beta_1) \gamma + \beta^2 \gamma^2}. \quad (8)
\]
So, we have:
Then the system (1)-(3) can be regarded as:

\[
\begin{cases}
\dot{\phi} + A(\phi, t) = F(\phi, t), & x \in \Omega, \quad t \geq \tau, \quad \tau \in \mathbb{R}, \\
\phi(\tau) = \phi_\tau = (u_{0r}, u_{0r} + \varepsilon u_{0r})^T.
\end{cases}
\] (9)

Where

\[
F(\phi, t) = \begin{pmatrix} 0 \\ -h(u) + q(x, t) \end{pmatrix},
\]

\[
A(\phi) = \begin{pmatrix} \varepsilon u - w \\ \Delta^2 u - \varepsilon(\gamma \Delta^2 - \xi I)u + (\gamma \Delta^2 - \xi I)w + g(w - \varepsilon u) \end{pmatrix}.
\] (10)

Next, we define a new weighted scalar product and norm in \( \mathbb{R} \) as:

\[
(\varphi, \psi)_E = k((u_1, u_2) + (w_1, w_2)), \quad \|\varphi\|_E = (\varphi, \varphi)_E,
\] (11)

For any \( \varphi = (u_1, w_1)^T, \psi = (u_2, w_2)^T \in E, \) where \( k \) is given by:

\[
k = \frac{4 + (\gamma \lambda_1^2 + \beta_1) \gamma + \frac{\beta_1^2}{\lambda_1^2}}{4 + 2(\gamma \lambda_1^2 + \beta_1) \gamma + \frac{\beta_1^2}{\lambda_1^2}} \in \left(\frac{1}{2}, 1\right].
\] (12)

**Lemma 2.** For \( \forall \varphi = (u, w)^T \in E, \) if (H3) hold, then,

\[
(\Lambda(\varphi), \varphi)_E \geq \chi \|\varphi\|_E^2 + \frac{\beta_1}{2} |v|^2 + \frac{\beta_2}{2} |w|^2,
\] (13)

Where

\[
\beta_2 \geq |\beta_1| \min \left\{ \frac{1}{\gamma}, \frac{\gamma \lambda_1^2 + \beta_1}{\lambda_1^2} \right\}.
\] (14)

And

\[
\chi = \frac{\gamma \lambda_1^2 + \beta_1}{\gamma_1 + \sqrt{\gamma_1 \gamma_2}},
\]

\[
\gamma_1 = 4 + (\gamma \lambda_1^2 + \beta_1) \gamma + \frac{\beta_1^2}{\lambda_1^2},
\]

\[
\gamma_2 = (\gamma \lambda_1^2 + \beta_1) \gamma + \frac{\beta_1^2}{\lambda_1^2}.
\] (15)

**Proof.** For \( \forall \varphi = (u, w)^T, \) let \( k = 1 - \varepsilon \gamma, \) we can conclude

\[
\begin{align*}
(\Lambda(\varphi), \varphi)_E &- \chi \|\varphi\|_E^2 - \frac{\gamma \lambda_1^2 + \beta_1}{2} |v|^2 \\
&= k \left( (\varepsilon u - w, u) + (\Delta^2 u - \varepsilon(\gamma \Delta^2 - \xi I)u + (\gamma \Delta^2 - \xi I)w + g(w - \varepsilon u), w) \\
&- \chi \left( k \|u\|^2 + |v|^2 \right) - \frac{\gamma \lambda_1^2 + \beta_1}{2} |v|^2 \\
&= k \varepsilon \|u\|^2 - k((w, u)) + (\Delta u, \Delta w) - \varepsilon \gamma((\Delta u, \Delta w) + \varepsilon \gamma (u, w) + \gamma \|u\|^2 - \varepsilon |v|^2 \\
&+ (g(w - \varepsilon u), w) - \chi k \|u\|^2 - \chi |v|^2 - \frac{\gamma \lambda_1^2 + \beta_1}{2} |v|^2.
\end{align*}
\]
\[
\begin{align*}
\geq & \ k \varepsilon \|w\|^2 - k((w, u)) + (1 - \varepsilon \gamma)(\Delta \alpha, \Delta w) + \varepsilon^2 (u, w) + \gamma \lambda_1^2 \|w\|^2 - \varepsilon \|w\|^2 \\
& + (g(w - \varepsilon u), w) - \chi k \|w\|^2 - \|w\|^2 - \varepsilon \lambda_1^2 + \frac{\beta_1}{2} \|w\|^2 \\
\geq & \ k \varepsilon \|w\|^2 - k \|w\|^2 - (\varepsilon \gamma - 1) \|w\|^2 + \varepsilon^2 (u, w) + \gamma \lambda_1^2 \|w\|^2 - \varepsilon \|w\|^2 \\
& + (g(w - \varepsilon u), w) - \chi k \|w\|^2 - \|w\|^2 - \varepsilon \lambda_1^2 + \frac{\beta_1}{2} \|w\|^2 \\
\geq & \ k \varepsilon \|w\|^2 - k \|w\|^2 - (\varepsilon \gamma - 1) \|w\|^2 + \varepsilon^2 (u, w) + \gamma \lambda_1^2 \|w\|^2 - \varepsilon \|w\|^2 \\
& + \beta \|w\|^2 - \varepsilon \beta \|w\|^2 - \gamma \lambda_1^2 \|w\|^2 - \chi k \|w\|^2 - \|w\|^2 - \varepsilon \lambda_1^2 + \frac{\beta_1}{2} \|w\|^2 \\
\geq & \ k \varepsilon \|w\|^2 - k \|w\|^2 - (\varepsilon \gamma - 1) \|w\|^2 + \varepsilon^2 (u, w) + \gamma \lambda_1^2 \|w\|^2 - \varepsilon \|w\|^2 \\
& + \beta \|w\|^2 - \varepsilon \beta \|w\|^2 - \gamma \lambda_1^2 \|w\|^2 - \chi k \|w\|^2 - \|w\|^2 - \varepsilon \lambda_1^2 + \frac{\beta_1}{2} \|w\|^2 \\
\geq & \ k \varepsilon \|w\|^2 + \left(\frac{\beta_1}{2} - \frac{3 \varepsilon}{2}\right) \|w\|^2 - \frac{\beta_1}{2} \|w\|^2 \\
\end{align*}
\]

Due to
\[ k \varepsilon \left(\gamma \lambda_1^2 + \beta_1 - 3 \varepsilon\right) \geq \frac{\varepsilon^3 \beta_1^2}{\lambda_1^2}, \]

We obtain
\[ (\Lambda(\varphi), \varphi)_E \geq \chi \|\varphi\|^2 + \frac{\lambda_1^2 + \beta_1}{2} \|\varphi\|^2. \]

The proof is completed.

**Lemma 3.** For \( \forall \varphi = (u, w) \in E \), if (H1)-(H3) hold, then, we have
\[ \|\varphi(t)\|^2 \leq \|\varphi(0)\|^2 \exp\left(-2\delta(t - \tau)\right) + \frac{\left(\|\varphi(t)\|^2 + c_1\right)^2}{2\delta(\lambda_1^2 + \beta_1)}. \] (16)

**Proof.** Take the scalar product \((\cdot, \cdot)_E\) of (9) with \(\varphi\), we have
\[ \frac{1}{2} \frac{d}{dt} \|\varphi\|^2_E + (\Lambda(\varphi), \varphi)_E = (F(\varphi, t), \varphi)_E. \] (17)

Where \(\varphi = (u, w) \in E\) be the solution of system (9). By (13),
\[ -(\Lambda(\varphi), \varphi)_E \leq -\delta \|\varphi\|^2_E - \frac{\lambda_1^2 + \beta_1}{2} \|\varphi\|^2, \]

Due to (H2) and (10),
\[ (F(\varphi, t), \varphi)_E = (-b(u) + q(x, t), w) \leq c_1 \|w\|^2 + \|q\|_{L_1} \|w\| \]
\leq \(\frac{c_1 + \|q\|_{L_1}}{\lambda_1^2 + \beta_1} + (\lambda_1^2 + \beta_1) \|w\|^2\),

Here \(\|q\|_{L_1} = \sup_{x, t} \|q(x, t)\|^2\). Through the simple calculation, we can have:
Applying the Gronwall’s inequality, we can conclude the absorbing property:

\[
\|\varphi(t)\| \leq \varphi(\tau) \exp(-2\delta(t-\tau)) + \left(\frac{\|q_0\| + c}{\delta(\gamma \lambda^2 + \beta)}\right)^2.
\]

Thus, the proof is completed.

It is easy to check that

\[
B_0 = \left\{(u, w) \in E : \|\varphi(t)\| \leq \left(\frac{\|q_0\| + c}{\delta(\gamma \lambda^2 + \beta)}\right)^2 \right\} \subset E,
\]

be a bounded uniformly absorbing set. Therefore, for any bounded subset \( B \subset E \), there exists \( t' = t'(\tau, B) \geq \tau \) such that \( U(t, \tau)B \subset B_0 \).

To prove the uniformly asymptotical compactness of the family of processes \( \{U(t, \tau)\} \) is a key to obtain the existence of a uniform attractor for the process. Here, we will show that the process \( \{U(t, \tau)\} \) possesses a uniformly attracting compact set, by using the intermediate Sobolev space.

**Theorem 1.** Assume the conditions (H1)-(H3) hold and \( q(x, t) \in L^2_\infty (R; H) \) be translation compact, and then the process \( \{U(t, \tau)\} \) corresponding to system (1)-(3) has the uniform attractor

\[
A = \bigcup_{q \in \mathcal{H}(q_0)} K_s(0),
\]

where the hull

\[
H(q_0) = \{q_0(x, t + h) : h \in R\}_{|_{L^2_\infty (R, H)}}
\]

is compact in \( L^2_\infty (R; H) \) and

\[
K_s(0) = \left\{Z(\cdot, 0) : \left[Z(x, t) \in K_s \right] \right\}
\]

is the section of the non-empty kernel \( K_s \) at the time \( t = 0 \).

**Proof.** We firstly verify the \((E \times H(q_0), E) - \) continuity of the family of processes. We consider the corresponding solutions \( \varphi_1(t) \) and \( \varphi_2(t) \) of (9) with the initial data \( \varphi_{1,0} \) and \( \varphi_{2,0} \in E \), respectively. Then, the difference \( \varphi = \varphi_1 - \varphi_2 \) satisfies:

\[
\frac{\partial \varphi(t)}{\partial t} + \Lambda(\varphi_1(t)) - \Lambda(\varphi_2(t)) = F(\varphi_1(t), t) - F(\varphi_2(t), t),
\]

(18)

Where

\[
F(\varphi_1(t), t) - F(\varphi_2(t), t) = \begin{pmatrix} 0 \\ -h(u_1) + h(u_2) + q_1(x, t) - q_2(x, t) \end{pmatrix}.
\]

Taking the scalar product \((\cdot, \cdot)_E \) of (18) with \( \varphi \), we have

\[
\frac{1}{2} \frac{d}{dt} \|\varphi\|^2 + (\Lambda(\varphi_1) - \Lambda(\varphi_2), \varphi)_E = (F(\varphi_1, t) - F(\varphi_2, t), \varphi)_E.
\]

(19)

Next, we estimate the right of (18) as follows:

\[
(F(\varphi_1, t) - F(\varphi_2, t), \varphi)_E = -h(u_1) + h(u_2) + q_1(x, t) - q_2(x, t, w)
\]

(20)

\[
\leq \frac{c}{\lambda^2} \|w\|^2 + \frac{h_1 - q_2x^2}{2(\gamma \lambda^2 + \beta)} + \frac{\gamma^2 \lambda^2 + \beta}{2} \|w\|^2.
\]

By lemma 2, we obtain:

\[
(\Lambda(\varphi_1) - \Lambda(\varphi_2), \varphi)_E \geq \chi \|\varphi\|^2 + \frac{\gamma^2 \lambda^2 + \beta}{2} \|w\|^2,
\]

(21)

Where
Thus,
\[
\frac{d}{dt} |\varphi|^2 \leq -2X |\varphi|^2 \left( (\gamma \lambda_1^2 + \beta_1) |w|^2 + \frac{2c_2}{\lambda_1} \|v\| |w| + \frac{|q_1 - q_2|^2}{\gamma \lambda_1^2 + \beta_1} \right) + \frac{|q_1 - q_2|^2}{\gamma \lambda_1^2 + \beta_1}.
\]

Furthermore,
\[
|\varphi|^2 \leq \left( \varphi_0 - \varphi_1 \right) \left( 1 + \frac{1}{\gamma \lambda_1^2 + \beta_1} \int |q_1(s) - q_2(s)|^2 \, ds \right) \exp \left( -2X \frac{2c_2}{\lambda_1 \sqrt{k}} (t - \tau) \right)
\]

Form the above results, we can conclude that the family of processes \( \{U(t, \tau), t \geq \tau\} \) is uniformly asymptotically compact, and \((E \times H(q_0), E)\) – continuous. That is, the process \( \{U(t, \tau)\} \) corresponding to system (1)-(3) has the uniform attractor and the hull \( H(q_0) \) is compact in \( L^2_{loc}(R; H) \) and \( K_0 \) is the section of the non-empty kernel \( K_0 \) at the time \( t = 0 \). Thus, we show the existence of uniform attractor.

3. Conclusion

This study was motivated by the observation of the asymptotic behavior of solutions for infinite dimensional dynamical systems. Unlike previous studies, the current paper has carefully examined the uniform attractors of a non-autonomous strongly damped beam equations. In our analytical framework, we use the semigroup theorem and the technique of decomposing solutions. In addition, we have attempted to introduce a certain two-parameter region. The results suggest that the system have the absorbing set, thereby producing the uniform attractors. The study of attractors is one of the central problems in the non-autonomous systems. The findings have been discussed by many mathematical physical workers. However, the existence of uniform attractors of non-autonomous beam equations seldom research. Therefore, it is worth being research.

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5. References


