Abstract

In this paper, exponential synchronization for stochastic complex networks with delays via impulsive control is investigated. We derive some simple but generic sufficient conditions for global exponential synchronization of the impulsive-control delayed dynamical complex networks based on Lyapunov stability theory, stochastic analysis theory as well as linear matrix inequality (LMI). Finally, the numerical simulation is provided to show the effectiveness of the theoretical results.

Keywords: Complex Networks Exponential Synchronization Stochastic Perturbations Impulsive control

1. Introduction

Over the past two decades, increasing interest has been shown to the study of complex networks, especially the investigation on the synchronization of complex networks has attracted a great deal of attention due to its potential applications in various fields, such as physics, mathematics, secure communication, engineering, automatic control, biology and sociology [1] - [9].

In order to make the synchronization conclusions obtained serve the practical society, many researchers in this field start to turn their eyes to probe into deeper and more realistic complex network models. It is this aim that urges those workers to make a thorough study of synchronization of delayed complex networks with stochastic perturbations.

Why do those mathematical workers consider the time delay and stochastic perturbations? First of all, time delays are ubiquitous in the real world. In complex dynamical networks, the inner time delay causes chaos, such as delayed neural networks and delayed Chua's circuit systems, etc; the outer time delay (coupling delay) cannot be ignored yet, because of communication and traffic congestion, etc. Therefore, time delays should be considered in order to simulate realistic networks.

In [10][11][12][13], the authors investigated synchronization of complex networks with coupling delays. Besides, uncertainties commonly exist in the real world, such as stochastic forces on the physical systems and noisy measurements caused by environmental uncertainties, so a stochastic behavior should be produced instead of a deterministic one [14]. In fact, transmitted signals between nodes of complex networks are unavoidably subject to stochastic perturbations from environment, which may cause information contained in these signals to be lost [15]. Therefore, stochastic perturbations have to be considered. In many existent literature, such as stochastic perturbations in [15][16][17] are all one-dimensional, which means that the signal transmitted by nodes is influenced by the same noise. In [14][18], the authors considered stochastic synchronization of coupled neural networks, in which noise perturbations are vector forms. Vector-form perturbation means that different nodes are influenced by different noise, and hence it is closer to our real world.

Moreover, impulsive phenomenon is widespread as transient mutation act in nature. There are impulsive phenomena in topology of complex dynamical networks, too. For example, the conjunction of nodes has impulsive effect when transmission signals are switched in networks. Impulsive control has attracted more interest due to its easy implementation in engineering control [19]. In [20], the

Based on the above analysis, in this paper, we study the synchronization of complex networks with delays and stochastic perturbations by impulsive control. To obtain our main results, we first formulate a kind of new complex network model with inner delay, non-delayed and delayed couplings and Wiener processes. By using the Lyapunov functional method, the stochastic stability analysis theory and linear matrix inequality technique, some novel sufficient conditions are derived to guarantee exponential synchronization of the complex networks.

The paper is organized as follows. In Sect. 2, a general model of stochastic complex network with both delayed dynamical nodes, delayed couplings and some preliminaries are given. In Sect. 3, some exponential synchronization criteria for such complex dynamical network are established. In Sect. 4, a numerical example for verifying the effectiveness of the theoretical results is provided. We conclude the paper in Sect. 5.

2. Preliminaries and model description

2.1 Notations

$\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively. The super script $T$ denotes the transpose of a matrix or vector, $\text{Tr}(\cdot)$ denotes the trace of the corresponding matrix and $I_n$ denotes an n-dimensional identity matrix and $A' = (A + A')/2$. For the square matrix $M$, the notation $M > 0$ ($< 0$) denotes $M$ is a positive-definite (negative-definite) matrix. If $A$ is a symmetric matrix, $\lambda_{\text{max}}(A)$ and $\lambda_{\text{min}}(A)$ are used to denote its largest and smallest eigenvalues, respectively. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, P)$ be a completed probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ which is right continuous and $\mathcal{F}_0$ contains all $P$-null sets. Let $w(t) = (w_1(t), w_2(t), \ldots, w_n(t))^T$ be an $m$-dimensional Brownian motion defined on probability space. Denoted by $C([-\tau, 0]; \mathbb{R}^n)$, the family of continuous function $\phi$ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\| = \sup_{t \in [-\tau, 0]}|\phi(s)|$. (Denote by $C^2([-\tau, 0]; \mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable, $C([-\tau, 0]; \mathbb{R}^n)$-valued stochastic variables $\xi = \{\xi(\theta); -\tau \leq \theta \leq 0\}$ such that $\int_{-\tau}^0 E \|\xi(s)\|^2 ds \leq \infty$, where $E$ stands for the correspondent expectation operator with respect to the given probability measure $\mathbb{P}$.

Consider a complex network consisting of $N$ identical nodes with nondelayed and delayed linear couplings and vector-form stochastic perturbations, which is described as

$$
dx_i(t) = \left[ f(t, x_i(t), x_i(t-\tau)) + \sum_{j=1,j\neq i}^N a_{ij} \Gamma(x_j(t) - x_i(t)) + \sum_{j=1,j\neq i}^N b_{ij} \Gamma x_j(t-\tau(t)) \right] dt 
+ \sigma_i(t, x(t), x(t-\tau), x(t-\tau)) dw_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^r$ represents the state vector of the $i$th node of the network; $f(t, x_i(t), x_i(t-\tau)) = [f_1(t, x_i(t), x_i(t-\tau)), f_2(t, x_i(t), x_i(t-\tau)), \ldots, f_n(t, x_i(t), x_i(t-\tau))]$ is a continuous vector-form function; $\Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)$ is an inner coupling configuration of the networks, satisfying $\gamma_j > 0, \quad j = 1, 2, \ldots, n$. $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ and $B = [b_{ij}] \in \mathbb{R}^{n \times n}$ are outer coupling matrices of the networks at time $t$ and $t - \tau$, respectively, satisfying $a_{ij} \geq 0$ for $i \neq j$, $a_{ii} = \sum_{j=1,j\neq i}^N a_{ij}$ and $b_{ij} \geq 0$ for $i \neq j$, $b_{ii} = \sum_{j=1,j\neq i}^N b_{ij}$; $\tau$ is the inner delay satisfying $\tau \geq 0$, and $\tau_c$ is the coupling delay.
satisfying $\tau_i \geq 0; \sigma_i(t,x(t),x(t-\tau)), x(t-\tau),\ldots,x(t-\tau_i)) = \sigma_i(t,x(t),\ldots,x(t-\tau_i))$.

$x(t-\tau),\ldots,x_i(t-\tau_i)) \in R^{n_i}$ and $w_i(t) = (w_{i1}(t),w_{i2}(t),\ldots,w_{in}(t))^T \in R^n$ is a bounded vector-form Weiner process. In this paper, we always assume that $w_i(t)$ and $w_j(t)$ are independent processes of one another for $i \neq j$, and $A$ is irreducible in the sense that there is no isolated node.

**Note:** In general, in the signal transmission process, the time delay affects only the variable that is being transmitted from one system to another. Therefore, it can be assumed that no delay exists in self-feedback. The delay coupling is inevitably recognized as $x_i(t-\tau) - x_j(t)$ [23][24][25].

The initial conditions associated with (1) are

$$x_i(s) = \xi_i(s), \quad -\bar{\tau} \leq s \leq 0, \quad i = 1,2,\ldots,N,$$

where $\bar{\tau} = \max\{\tau_i, \tau_j\}, \xi_i \in C^0_{\bar{\tau}}([-\bar{\tau}, 0], R^n)$ with the norm $||x_i|| = \sup_{s \in [-\bar{\tau},0]} \xi_i$. 

In the case that system (1) reaches synchronization, i.e. $x_i(t) = x_j(t) = \cdots = x_N(t) = s(t)$, we have the following synchronized state equation:

$$ds(t) = f(t,s(t),s(t-\tau))dt$$

where $i = 1,2,\ldots,N$. Obviously, the synchronization state $s(t)$ is uniform. Therefore, the assumption $b_{11} = b_{22} = \cdots = b_{NN}$ must be imposed.

In order to achieve the synchronization, the impulsive pinning controllers are added to a part of its nodes. Without loss of generality, let the first $l$ nodes be controlled. Thus the pinning controlled network can be described by

For $t \neq kT$, $k = 1,2,\ldots$

$$dx_i(t) = \{f(t,x_i(t),x(t-\tau)) + \sum_{j=1, j \neq i}^{N} a_{ij} \Gamma(x_j(t) - x_i(t)) + \sum_{j=1, j \neq i}^{N} b_{ij} \Gamma(x_j(t-\tau)) - x_i(t-\tau)) \}dt + \sigma_i(t,x(t),x(t-\tau),x(t-\tau_i))dw_i(t), \quad i = 1,2,\ldots,N,$$

(3)

For $t = kT$, $k = 1,2,\ldots$

$$\begin{cases}
x_i(t_k) = x_i(t_k^-) - \epsilon_i(x_i(t_k^-) - s(t_k)), & i = 1,2,\ldots,l, \\
x_i(t_k) = x_i(t_k^-), & i = l+1, l+2,\ldots,N,
\end{cases}$$

(4)

where $\epsilon_i > 0$ ($i = 1,2,\ldots,l$) are called control gains and denote $\Xi = \text{diag}\{\epsilon_1, \epsilon_2,\ldots,\epsilon_l, 0,\ldots,0\} \in R^{n}$; $T$ is the control period, $\delta \leq T$ is a positive constant called the control width (control duration) and denote $\theta = \delta / T$; and $(k = 0,1,\ldots)$. Define the synchronization errors as $e_i(t) = x_i(t) - s(t)$ ($i = 1,2,\ldots,N$), the error system is derived as following:

For $t \neq kT$, $k = 1,2,\ldots$

$$de_i(t) = \{f(t,x_i(t),x(t-\tau)) - f(t,s(t),s(t-\tau)) + \sum_{j=1}^{N} a_{ij} \Gamma e_j(t) + \sum_{j=1}^{N} b_{ij} \Gamma e_j(t-\tau)) \}dt$$

$$+ \sigma_i(t,x(t),x(t-\tau),x(t-\tau_i))dw_i(t), \quad i = 1,2,\ldots,N,$$

(5)

For $t = kT$, $k = 1,2,\ldots$
Definition 1. The complex network (6) is said to be exponentially synchronized if the trivial solution of system (5) s.t.

\[
\sum_{i=1}^{N} E \left\| e_i(t, t_0, \xi) \right\|^2 \leq K e^{-\kappa t},
\]

where \( K > 0 \) and \( \kappa > 0 \) for any initial data \( \xi \in \mathbb{R}^n \).

Definition 2. [14] A continuous function \( f(t, x, y) : [0, +\infty) \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) is said to be in the QUAD function class, denoted as \( f \in \text{QUAD}(P, \Delta, \eta, \zeta) \), for given matrix \( \Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_N) \), if there exists a positive definite diagonal matrix \( P = \text{diag}(p_1, p_2, \ldots, p_n) \), a diagonal matrix \( \Delta = \text{diag}(\delta_1, \delta_2, \ldots, \delta_N) \) and constants \( \eta > 0, \zeta > 0 \), such that \( f \) satisfies the following condition:

\[
(x - y)^T P((f(t, x, z) - f(t, y, w)) - \Delta \Gamma(x - y)) \leq -\eta(x - y)^T (x - y) + \zeta(z - w)^T (z - w)
\]

for all \( x, y, z, w \in \mathbb{R}^n \).

Note: The function class QUAD includes almost all the well-known chaotic systems with delays or without delays such as Lorenz system, Rössler system, Chen system, delayed Chua’s circuit as well as logistic delay differential system, delayed Hopfield neural networks and delayed CNNs, and so on. We shall simply write

\[
\hat{p} = \max\{p_1, p_2, \ldots, p_N\}, \quad \hat{\nu} = \min\{p_1, p_2, \ldots, p_N\}, \quad \delta = \max\{\delta_1, \delta_2, \ldots, \delta_N\}.
\]

The following assumptions will be used throughout this paper in establishing our synchronization conditions.

**H1** \( b_{11} = b_{22} = \ldots = b_{NN} = \overline{b} \)

**H2** Denote \( \sigma(t, e(t), e(t - \tau), e(t - \tau_N)) = \sigma(t, e_1(t), \ldots, e_N(t), e_1(t - \tau), \ldots, e_1(t - \tau_N), e_2(t - \tau), \ldots, e_N(t - \tau_N)) \). There exist appropriate-dimension positive definite constant matrices \( Y_{1i}, Y_{1j} \) and \( Y_{1i} \) for \( i = 1, 2, \ldots, N \), such that

\[
\text{Tr}[\sigma(t, e(t), e(t - \tau), e(t - \tau_N)) Y_{1i}, Y_{1j} e(t - \tau)] \leq \sum_{i=1}^{N} e_i(t)^T Y_{1i} e_i(t) + \sum_{i=1}^{N} e_i(t - \tau)^T Y_{1i} e_i(t - \tau) + \sum_{i=1}^{N} e_i(t - \tau_N)^T Y_{1i} e_i(t - \tau_N).
\]

**Lemma 1.** [26][Itô formula] Consider an \( n \)-dimensional stochastic differential equation

\[
dx(t) = f(t, x(t), x(t - \tau)) \, dt + \sigma(t, x(t), x(t - \tau)) \, d\omega(t)
\]

Let \( C^{21}(\mathbb{R} \times \mathbb{R}^n; \mathbb{R}) \) denote the family of all nonnegative functions \( V(t, x) \) on \( \mathbb{R} \times \mathbb{R}^n \), which are twice continuously differentiable in \( x \) and once differentiable in \( t \). If \( V \in C^{21}(\mathbb{R} \times \mathbb{R}^n; \mathbb{R}) \), define an operator \( LV \) form \( \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \) to \( \mathbb{R} \) by

\[
LV(t, x, y) = V_t(t, x) + V_x(t, x) f(t, x, y) + \frac{1}{2} \text{Tr}[\sigma(t, x, y)^T V_{xx} \sigma(t, x, y)],
\]

where \( V_t(t, x) = \partial V(t, x) / \partial t \), \( V_x(t, x) = (\partial V(t, x) / \partial x_1, \ldots, \partial V(t, x) / \partial x_n) \), \( V_{xx} = (\partial^2 V(t, x) / \partial x_i x_j)_{i,j} \). If \( V \in C^{21}(\mathbb{R} \times \mathbb{R}^n; \mathbb{R}) \), then for any \( \infty > t > t_0 \geq 0 \),
as long as the expectations of the integrals exist.

3. Main results

In this section, we present the synchronization criteria for delays complex networks with stochastic perturbations via impulsive pinning.

Theorem 1: Suppose the assumption H1 and H2 hold and $f \in \text{QUAD}(P, \Delta, \eta, \zeta)$. If there exist positive constants $\alpha$ and $\beta$, such that

$$\begin{bmatrix} A' + \delta I_N - \alpha I_N & B / 2 \\ B^T / 2 & -\beta I_N \end{bmatrix} \leq 0,$$

(9)

$$\bar{\sigma}(1 + b\tau e^{\gamma^2} + c\tau e^{\gamma^2}) < e^{\gamma^2},$$

(10)

$$\bar{\tau} \leq t_0, \quad \bar{\tau} \leq (1-\theta)T,$$

(11)

where $\gamma > 0$ is the smallest root of the equation

$$\gamma - a + b e^{\gamma} = 0,$$

$$a = \frac{\lambda_{\max}(-2\eta I_n + \tilde{p} \sum_{i=1}^{N} Y_{ii} + 2\alpha P T)}{\tilde{p}},$$

$$b = \frac{\lambda_{\max}(\sum_{i=1}^{N} P Y_{ii} + 2\zeta I_N)}{\tilde{p}},$$

$$c = \frac{\lambda_{\max}(\sum_{i=1}^{N} P Y_{ii} + 2\beta P T)}{\tilde{p}},$$

Then the solution $x_1(t), x_2(t), \ldots, x_N(t)$ of system (6) is globally exponentially synchronized to $s(t)$.

Proof. Let $\tilde{e}^k(t) = (e_{1k}(t), e_{2k}(t), \ldots, e_{nk}(t))^T, \quad k = 1, 2, \ldots, n$. Define a Lyapunov function

$$V(t, e(t)) = V(t) = \frac{1}{2} \sum_{i=1}^{N} (\tilde{e}^k(t))^T P \tilde{e}^k(t).$$

When $t \neq t_i$, according to Lemma 1, we have

$$LV(t, e(t)) = \sum_{i=1}^{N} \tilde{e}^k(t)^T P \{ f(t, x(t), x(t - \tau)) - f(t, s(t), s(t - \tau)) + \sum_{j=1}^{N} a_{ij} \Gamma e_j(t) + \sum_{j=1}^{N} b_{ij} \Gamma e_j(t - \tau_j) \}$$

$$+ \frac{1}{2} \text{Tr} \{ \sigma_i(t, x(t), x(t - \tau)) (x(t - \tau)) \} P \sigma_i(t, x(t), x(t - \tau), x(t - \tau)) \}$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \text{Tr} \{ \sigma_i(t, x(t), x(t - \tau)) x(t - \tau) \} P \sigma_i(t, x(t), x(t - \tau), x(t - \tau)) \}$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \text{Tr} \{ \sigma_i(t, x(t), x(t - \tau)) x(t - \tau) \} P \sigma_i(t, x(t), x(t - \tau), x(t - \tau)) \}$$

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From condition (9), we obtain
\[
LV(t,e(t)) \leq aV(t) + bV(t - \tau) + cV(t - \tau).
\]

When \( t = t_k \), we have
\[
V(t_k,e(t_k)) = \frac{1}{2} \sum_{i=1}^{N} e_i(t_k)^T P e_i(t_k)
\]
\[
\leq \frac{1}{2} \sum_{i=1}^{N} (1 + \epsilon_i) e_i(t_k)^T P e_i(t_k)
\]
\[
= \frac{1}{2} \sum_{i=1}^{N} (1 + \epsilon_i) e_i(t_k)^T P e_i(t_k) + \frac{1}{2} \sum_{i=1}^{N} e_i(t_k)^T P e_i(t_k)
\]
\[
\leq EV(t_k) \quad (\exists \Xi \in [\min\{(1 + \epsilon_i)^2, 1\}, \max\{(1 + \epsilon_i)^2, 1\}]) \quad (12)
\]

Define
\[
W(t) = W(t,e(t)) = e^tV(t,e(t)),
\]
For \( t \neq t_k \), we computer the operator
\[
LW(t,e(t)) = e^t\gamma V(t) + L V(t)
\]
\[
\leq e^t\gamma V(t) + aV(t) + bV(t - \tau) + cV(t - \tau).
\]

Therefore, by the generalized Itô formula, we have for any \( t > t_0 \geq 0 \),
\[
e^tEV(t) = e^{t_0}EV(t_0) + \int_{t_0}^{t} e^{s-t}LW(s)ds.
\]

When \( t \in (t_k, t_{k+1}] \), we have
\[
e^tEV(t) \leq e^{t_0}EV(t_0) + \int_{t_0}^{t} e^{s-t} [\gamma V(s) + aV(s) + bV(s - \tau) + cV(s - \tau)]ds
\]
\[
\leq \frac{P}{2} \sum_{i=1}^{N} \| \xi_i \|^2 + (\gamma + a) \int_{t_0}^{t} e^{s-t}EV(s)ds + be^{t_0} \int_{t_0}^{t} e^{s-t}EV(s - \tau)ds
\]
\[
+ ce^{t_0} \int_{t_0}^{t} e^{s(t - \tau)}EV(s - \tau)ds.
\] \quad (13)

By changing variable \( s - \tau = u \), we have
\[
\int_{t_0}^{t} e^{s(t - \tau)}EV(s - \tau)ds = \int_{t_0}^{t} e^{s(t - \tau)}EV(u)du
\]
\[
\leq \frac{P}{2} \sum_{i=1}^{N} \| \xi_i \|^2 + \int_{t_0}^{t} e^{s-t}EV(u)du.
\] \quad (14)
By changing variable \( s - \tau_c = u \), we have
\[
\int_{0}^{t} e^{(s - \tau_c)} \mathcal{E} V(s - \tau_c) ds = \int_{-\tau_c}^{t} e^{\alpha u} \mathcal{E} V(u) du
\]
\[
\leq \frac{p}{2} \tau_c \sum_{i=1}^{N} E \| \xi_i \|^2 + \int_{0}^{t} e^{\alpha u} \mathcal{E} V(u) du.
\]
(15)

Substituting the equations (14) and (15) into the equation (13), we get
\[
ed^{\alpha t} \mathcal{E} V(t) \leq \frac{p}{2} (1 + b\tau e^{\alpha T} + c\tau e^{\alpha T}) \sum_{i=1}^{N} E \| \xi_i \|^2 e^{(\gamma + a + be^{\alpha T} + ce^{\alpha T} \tau) t} \]
\[
+ (\gamma + a + be^{\alpha T} + ce^{\alpha T}) \int_{0}^{t} e^{\alpha u} \mathcal{E} V(u) du.
\]
By using the Gronwall inequality, we get
\[
ed^{\alpha t} \mathcal{E} V(t) \leq \frac{p}{2} (1 + b\tau e^{\alpha T} + c\tau e^{\alpha T}) \sum_{i=1}^{N} E \| \xi_i \|^2 e^{(\gamma + a + be^{\alpha T} + ce^{\alpha T} \tau) t}.
\]
(16)

When \( t \in (t_i, t_{i+1}] \), we have
\[
ed^{\alpha t} \mathcal{E} V(t) \leq e^{\gamma T} \mathcal{E} V(t_i^+) + E \int_{t_i}^{t} e^{\alpha u} \mathcal{E} [Y V(s) + aV(s) + bV(s - \tau) + cV(s - \tau_c)] ds
\]
\[
\leq e^{\gamma T} \mathcal{E} V(t_i^+) + b\tau c \int_{-\tau_c}^{t_i} e^{\alpha u} \mathcal{E} V(s) ds + c\tau e^{\alpha T} \int_{-\tau_c}^{t_i} e^{\alpha u} \mathcal{E} V(s) ds
\]
\[
+ (\gamma + a + be^{\alpha T} + ce^{\alpha T}) \int_{0}^{t_i} e^{\alpha u} \mathcal{E} V(u) du
\]
\[
\leq \left( \frac{p}{2} (1 + b\tau e^{\alpha T} + c\tau e^{\alpha T}) \right) \sum_{i=1}^{N} E \| \xi_i \|^2 e^{(\gamma + a + be^{\alpha T} + ce^{\alpha T} \tau) t}.
\]

Similarly, we can prove that when \( t \in (t_i, t_{i+1}] \), we have
\[
ed^{\alpha t} \mathcal{E} V(t) \leq (\mathcal{E} \frac{p}{2} (1 + b\tau e^{\alpha T} + c\tau e^{\alpha T}) \sum_{i=1}^{N} E \| \xi_i \|^2 e^{(\gamma + a + be^{\alpha T} + ce^{\alpha T} \tau) t}.
\]

By the condition (10), we obtain
\[
\sum_{i=1}^{N} E \| e_i(t) \|^2 \leq K e^{-\kappa t},
\]
where \( K = \frac{p}{2p} (1 + b\tau e^{\alpha T} + c\tau e^{\alpha T}) (1 + b\tau e^{\alpha T} + c\tau e^{\alpha T}) \sum_{i=1}^{N} E \| \xi_i \|^2 e^{(\gamma + a + be^{\alpha T} + ce^{\alpha T} \tau) t} \) and \( \kappa > 0 \).

The proof is completed.

\textbf{Note:} In Theorem 1, the coupling matrix \( B \) is not necessarily a symmetrical or irreducible matrix. The conditions of Theorem 3 imply that the system (6) is exponentially stabilized with exponential degree \( \kappa \) which is dependent on \( T \).

\textbf{Note:} From conditions (9) in Theorem 1, we can determine the control strength \( e_i \), and the number of controlled nodes \( I \). (In [27][28], the authors found the relationship between coupling strength and the number of pinning nodes of a network with a fix network structure.)

4. Numerical simulations

In this section, we will provide a numerical simulation to verify the theorem given in the previous section.
Consider the following chaotic delayed neural networks:
\[
dx(t) = [-C x(t) + Af(x(t)) + Bg(x(t-\tau))] dt,
\]
where \( f(x) = g(x) = \tanh(x) \), \( \tau = 1 \),

\[
C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & -0.1 \\ -5 & 4.5 \end{bmatrix}, \quad B = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{bmatrix}.
\]

Taking \( P = \text{diag}\{1,2\} \) and \( \Delta = \text{diag}\{5,11,5\} \), then we have \( \eta = 0.15 \), \( \zeta = 3.25 \). Hence the condition (7) is satisfied [14].

In order to verify our results, we consider the following complex network:
\[
dx_i(t) = \left\{ f(x_i(t),x_i(t-\tau)) + \sum_{j=1}^{10} a_{ij} \Gamma(x_j(t)-x_i(t)) + \sum_{j=1}^{10} b_{ij} \Gamma(x_j(t-\tau_i)-x_i(t)) \right\} dt
\]

\[
+ \sigma_i(t,x_i(t),x_i(t-\tau),x_i(t-\tau_i))dw_i(t), \quad i = 1,2,\ldots,10,
\]

where
\[
A = 16 \times \begin{bmatrix} -5 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & -5 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -5 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & -5 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & -5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & -5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & -5 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ -5 \end{bmatrix},
\]

\[
B = 0.01 \times \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \\ -5 \end{bmatrix},
\]

\[
\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \tau_\epsilon = 0.1 \frac{e'}{1+e'}
\]

and
\[
\sigma_i(t,x_i(t),x_i(t-\tau),x_i(t-\tau_i)) = 0.1 \times \text{diag}\{x_{i1}(t)-x_{i11}(t),x_{i2}(t)-x_{i21}(t)\}.
\]

Through computation, we get \( \tau = 1 \), \( \tau_\epsilon = 0.1 \), \( Y_y = 0.01I_i \) for \( i = 1,2,\ldots,N \), and \( j = 1,2 \). Let the set of controlled nodes \( J = \{1,2,3,4,5\} \), and the control strength \( \epsilon_i = 90 \) for \( i = 1,2,3,4,5 \). By using
the Matlab LMI toolbox, we can obtain the following solution for conditions (9)-(11) of Theorem 1
\[ a = 80.1500, \ b = 6.5300, \ c = 0.0307, \ \alpha = 40.0000, \ \beta = 0.0002, \ \text{and} \ T = 8.8655 \ \text{and} \ \theta = 0.7. \]

The initial conditions of the numerical simulations are as follows: \( x_0(t) = \xi_j, \ i = 1, 2, \ldots, 10, \ j = 1, 2, \) for all \( t \in [-1, 0], \) where \( \xi_j \) are chosen randomly in \([-4, 4]\). By using the Euler-Maruyama method, the simulation are carried out using \( \Delta = 0.01. \) The trajectories of the system (18) by impulsive pinning control gains are shown in Figure 1. Figure 2 shows the time evolutions of synchronization errors with impulsive pinning control.

![Figure 1: Trajectories of \( x_{i1} \) and \( x_{i2} \) for System (18).](image)

![Figure 2: Trajectories of \( e_{i1} \) and \( e_{i2} \) for System (18).](image)

5. Conclusion

In this paper, we investigated the synchronization problem for stochastic complex networks with nondelay and delay coupling. In particular, we achieved global exponential synchronization by applying an impulsive pinning control scheme to a small fraction of nodes and derived sufficient conditions for the global exponential stability of synchronization. Finally, for clarity of exposition, some numerical examples were considered to illustrate the theoretical analysis. Our results improved and extended some existing results.

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7. References