Research on the Performance of the Time Delay Estimation in the Multi-path Wireless Channel based on Super-Resolution Algorithms

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Abstract

When the time difference of arrival between two paths in the multi-path wireless channel environment is less than the reciprocal of the signal bandwidth, the cross-correlation method to estimate the time delays of different paths will result in large error. In this case, super-resolution algorithms play an important role. In this paper, we give detailed descriptions of three super-resolution algorithms for time delay estimation, and study the performance of them under different signal to noise ratios and different sampling bandwidths. The simulation results show that the performance of MUSIC algorithm is much more inferior to the ESPRIT and GEESE algorithms, and its time complexity is also very high; although the performances of ESPRIT and GEESE are almost the same, the GEESE is somewhat superior to the ESPRIT in the time complexity. So GEESE algorithm is the preferred in the time delays estimation in multi-path wireless channel.

Keywords: Time Delay Estimation, Multi-Path Channel, Super-Resolution Algorithm

1. Introduction

Time delay estimation is a hot research topic in the international field of signal processing, and it is usually used to get the time offset of a signal compared with some referenced signal. The time delay estimation technique not only has important theoretical significance, but also has import application value, and could be used in biomedicine, sonar and wireless communication, and has wide application. For example, to locate an object accurately based on the wireless sensor networks [1], the distance measure based on time delay is the best way.

The main target of the time delay estimation is to improve the time resolution and to reduce the time complexity. As far as we know, all these existing algorithms can be classified into two categories: one is for the wide-band signal, and the other is for the narrow-band signal. Because the wide-band signal has a good capacity to overcome the multi-path interference, the concerned algorithms are concentrating on the time delay between two single paths. However, for the narrow-band signal, it is hard to distinguish two closed arrived signals, and delay estimation algorithm for wideband signal is no longer suitable for such an occasion, so researchers are more concerned on estimating the time delay of different paths.

For the time delay estimation of wide-band signal, algorithms based on the pattern matching are the most common and classical, in which the matching degree of two signals is the function of the time delay difference. Different matching functions lead to different time delay estimation algorithms, which include cross correlation function [2][3], the square error sum function [4], the normalized cross correlation function [5], and so on. However, the time resolutions of all these algorithms are determined by sampling interval, and high sampling rate will produce high resolution. However, we can not relay on increasing the sampling rate, which require expensive A/D sampling chip, to improve the time resolution. In order to solve this problem, we could adopt the interpolation technology [6] or the fractional delay filter technology [7]. Besides, there are some other kinds of algorithms, such as the minimum entropy [8], the Hilbert transformation [9], and the quadrature phase detection [10], and so on.
Because it is hardly to distinguish two arrival signals in different paths with the time delay difference less than the reciprocal of the source signal bandwidth [11], traditional algorithms for wideband signal time delay estimation are no longer suitable for narrowband signal. Super-resolution of time delay estimation of the narrow-band signal becomes the research focus [12]. In this case, some super-resolution algorithms which include MUSIC [13], ESPRIT [14], GEESE [15] and their improve algorithms play an important role. However, as far as we know, these algorithms have not been compared and analyzed when they are used to estimate the time delays of the multi-path wireless signal. So in this paper, we first give the detailed descriptions of three super-resolution algorithms for time delay estimation, and then study the performance of them under different signal to noise ratios and different sampling bandwidths.

The rest of this paper is organized as follows: section 2 is the system model, which represents the discrete frequency domain channel response as a matrix format; section 3 gives the details of super-resolution algorithms for time delay estimation; section 4 compares and analyzes the performances of the three algorithms; and this paper is concluded in section 5.

2. System Model

The time domain impulse response of a multi-path wireless channel is usually formulated as [16]:

\[ h(t) = \sum_{d=0}^{D-1} \alpha_d \delta(t - \tau_d) \]  

(1)

where \( D \) is the number of paths, \( \tau_d \) and \( \alpha_d \) is the time delay and complex amplitude of the \( d \) th path respectively, and \( \delta \) is the Dirac pluses. So the frequency domain channel impulse response is formulated as:

\[ h(f) = \sum_{d=0}^{D-1} \alpha_d e^{-j2\pi f \tau_d} \]  

(2)

The frequency domain response could be measured by the vector network analyzer [17].

After getting the frequency domain channel response, we uniformly sample it from \( f_L \) to \( f_H \) with the sampling frequency \( f_0 = (f_H - f_L) / (N - 1) \), and will obtain \( N \) samples. The whole sample bandwidth is \( f_H - f_L \). Then the discrete frequency domain response can be represented as:

\[ Y(k) = h(k) + W[k] = \sum_{d=0}^{D-1} \alpha_d e^{-j2\pi (f_0 + (k-1)f_0) \tau_d} + W(k) \]  

(3)

where \( W(k) \) is additive white Gaussian noise (AWGN) with zero mean and variance \( \delta_W^2 \). We then rewrite (3) in a matrix format as follows:

\[ Y = Ta + W \]  

(4)

\[ Y = [Y(0) \ Y(1) \cdots \ Y(N-1)]^T \]  

(5)

\[ T = [e(\tau_0), e(\tau_1), \cdots, e(\tau_{D-1})] \]  

(6)
We assume that $\mathbf{W}(k)$ is AWGN and the elements in the vector $\mathbf{W}$ are independent, so $\mathbf{W}$ satisfies $E(\mathbf{WW}^T) = \sigma_w^2 \mathbf{I}$, where $\mathbf{I}$ is an identity matrix.

3. Super-resolution algorithms

The super-resolution algorithms mainly refer to MUSIC, ESPRIT, GEESE and their improved algorithms. In this section, we will introduce the main processes of above three algorithms to estimate the time delays of different paths based on the channel frequency response.

3.1. MUSIC Algorithm

MUSIC is based on the principle that the signal subspace and noise subspace are orthogonal. It constructs the two subspaces utilizing the samples, design the array manifold and search in the one-dimensional interval to minimize the distance between the array manifold vector to the signal subspace or maximize the distance between the array manifold vector to the noise subspace, and finally obtain the time delays.

The implementation of MUSIC [18] is as follows:

S1. Constructs the auto-correlation matrix $\mathbf{R}_{yy} = E(\mathbf{Y}\mathbf{Y}^H)$ according to the sample vector $\mathbf{Y}$, where $^H$ denotes the conjugate transpose. We assume that the dimension of $\mathbf{Y}$ is $m$.

S2. Perform the eigenvalue decomposition of the matrix $\mathbf{R}_{yy}$, and sort the eigenvalues from largest to the smallest, and obtain the corresponding eigenvectors.

S3. Estimate the number of multi-path $D$ [19];

S4. Construct the signal subspace $E_S$ using the eigenvectors corresponding to the maximum $D$ eigenvalues; and construct the noise subspace $E_N$ using the eigenvectors corresponding to the the remaining $m-D$ eigenvectors.

S5. Take one-dimension search for the expression $\mathbf{e}^H(\tau)E_N\mathbf{e}(\tau)$, find the $D$ smallest values, and the corresponding $\tau$ is the time delays of the $D$ paths.

3.2. ESPRIT Algorithm

ESPRIT [20] is based on the principle that the signal subspace has the rotation invariant feature. In order to obtain the time differences of arrival, we need to construct an auto-correlation matrix and a cross-correlation matrix, and to find the relationship between them. ESPRIT has two main forms: one is based on the least square criterion, which is called LS-ESPRIT, and the other is based on the total least square criterion, which is called TLS-ESPRIT. Because the performance of TLS-ESPRIT is far superior to the LS-ESPRIT, we only introduce the TLS-ESPRIT to time delay estimation.

The implementation of TLS-ESPRIT is as follows:

S1. Constructs the auto-correlation matrix $\mathbf{R}_{yy} = E(\mathbf{Y}\mathbf{Y}^H)$ according to the sample vector $\mathbf{Y}$, where $^H$ denotes the conjugate transpose. We assume that the dimension of $\mathbf{Y}$ is $m$.

S2. Perform the eigenvalue decomposition of the matrix $\mathbf{R}_{yy}$, and sort the eigenvalues from largest to the smallest, and obtain the corresponding eigenvectors.
S3. Estimate the number of multi-path $D$;

S4. Construct the signal subspace $E_S$ using the eigenvectors corresponding to the maximum $D$ eigenvalues;

S5. Form a $m \times m$ identity matrix $\Sigma$, and create a $(m-1) \times m$ matrix $\Sigma_F$ with the first $m-1$ rows of $\Sigma$, and a $(m-1) \times m$ matrix $\Sigma_L$ with the last $m-1$ rows of $\Sigma$;

S6. Create a $(m-1) \times 2D$ matrix $U = [\Sigma_F E_s, \Sigma_L E_s]$, and perform the singular value decomposition $U = SDV^{-1}$;

S7. Partition the $2D \times 2D$ matrix $V$ into four $D \times D$ block matrix $V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$, and assume $\Phi = -V_{12}V_{22}^{-1}$;

S8. Obtain the $D$ eigenvalues $\lambda_i, i = 1, \ldots, D$ of matrix $\Phi$, and we could get the time delays of the multi-path signal $\tau_i, i = 1, \ldots, D$ using expression (10):

$$\tau_i = \text{angle}(\lambda_i)/(2\pi f_0)$$

Where $\text{angle}(x)$ denotes the angle degree of the complex number $x$.

3.3. GEESE Algorithm

GEESE algorithm [17] is based on the generalized eigenvalues utilizing the signal subspace eigenvectors, and can be seen as an extension of the MUSIC algorithm. The implementation of GEESE is as follows:

S1. Constructs the auto-correlation matrix $R_{YY} = E(YY^H)$ according to the sample vector $Y$, where $H$ denotes the conjugate transpose. We assume that the dimension of $Y$ is $m$.

S2. Perform the eigenvalue decomposition of the matrix $R_{YY}$, and sort the eigenvalues from largest to the smallest, and obtain the corresponding eigenvectors

S3. Estimate the number of multi-path $D$;

S4. Construct the signal subspace $E_S$ using the eigenvectors corresponding to the maximum $D$ eigenvalues;

S5. create a $(m-1) \times m$ matrix $E_i$ with the first $m-1$ rows of $E_S$, and a $(m-1) \times m$ matrix $\Sigma_2$ with the last $m-1$ rows of $E_S$;

S6. Perform the generalized eigenvalue decomposition of the matrix pencil $(E_i, E_2)$, obtain the generalized eigenvalues $\lambda_i, i = 1, \ldots, D$, and we can also get the time delays of the multi-path signal $\tau_i, i = 1, \ldots, D$ using expression (10).

4. Performance analysis and comparison

In this section we will compare the performances of above three super-resolution algorithms for estimating the time delays of different paths, and also analyze the time complexities of all these algorithms.
4.1. Performance analysis

The simulation parameters are as follows: the starting sampling frequency $f_s$ is 5GHz, the sampling interval $f_s$ is 2MHz; the number of paths is 6, their time delays are 20ns, 30ns, 40ns, 50ns, 60ns and 70ns individually and their complex amplitudes are 1, 1/2, 1/4, 1/8, 1/16 and 1/32 individually.

The standard variance of time delays is computed by $\sqrt{\sum_{d=0}^{D-1} (\hat{\tau}_d - \tau_d)^2}$, where $\hat{\tau}_d$ and $\tau_d$ represent the estimation value and real value of the $d$ th path.

Figure 1 shows the results of standard variances of time delays when the whole sampling bandwidth is 300MHz and the signal to noise ratio at the receiver changes from 5dB to 50dB. Obviously, the variances will reduce when the SNR increases, which is consistent with the simulation results. However, from Figure 1, we could see that the performance of MUSIC is far inferior to the ESPRIT and GEESE, and the performances of ESPRIT and GEESE are almost the same.

![Figure 1. Standard Variance under different SNR](image)
Figure 2 shows the results of standard variances of time delays when the SNR is 30dB, and the whole sampling bandwidth changes from 250MHz to 350MHz. From Fig.2, we could see that as the bandwidth increases, the performances of the three algorithms change for the better. This is because the increase of the bandwidth brings the increase of the number of samples and thereby the impact of noise is reduced. Similarly, the performance of MUSIC is far inferior to the ESPRIT and GEESE, and the performances of ESPRIT and GEESE are almost the same, which is the same to the Figure 2.

4.2. Time complexity analysis

After observing the implementation processes of the three algorithms, we could find that the first four steps are almost the same, which needs to obtain the auto-correlation matrix of the samples, and then need a matrix eigenvalue decomposition, so as to obtain the signal subspace and noise subspace. And thereafter, MUSIC requires a global search on a one dimensional space, and the step determines the search time the final estimation accuracy; ESPRIT algorithm requires one singular value decomposition and one eigenvalue decomposition; however, GEESE needs only one generalized eigenvalue decomposition.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Running Time of S1 (Second)</th>
<th>Running Time of S2 (Second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MUSIC</td>
<td>13620.6</td>
<td>15902.5</td>
</tr>
<tr>
<td>ESPRIT</td>
<td>40.4</td>
<td>45.2</td>
</tr>
<tr>
<td>GEESE</td>
<td>39.6</td>
<td>44.3</td>
</tr>
</tbody>
</table>

Table 1 shows the running time of the three algorithms. S1 represents the 100 times running time when the SNR changes from 5dB to 50dB and the whole sampling bandwidth is 300MHz, and the simulation results correspond to Fig.1; S2 represents the 100 times running time when the whole sampling bandwidth changes from 250MHz to 350MHz and the SNR is 30dB, and the simulation results correspond to Fig.2. From table 1, we could find that the running time of MUSIC is much larger than the ESPRIT and GEESE, and the running time of GEESE is slightly less than the ESPRIT. From the previous analysis, it is easy to understand that MUSIC requires a one-dimensional global search,
lending to the simulation time is too long. GEESE is less than the ESPRIT for a singular value decomposition step, so it’s running time is somewhat less than the ESPRIT.

5. Conclusions

In this paper, we introduce three super-resolution algorithms for time delay estimation in multi-path wireless channel. We compare the performances of the three algorithms under different signal to noise ratios and different sampling bandwidths. Simulation and analysis results show that the MUSIC algorithm results in the worst performance and has large time complexity; although the ESPRIT algorithm has the same performance with the GEESE algorithm, the time complexity of the latter is somewhat less than the former. The GEESE is the preferred super-resolution algorithm to estimate the time delays of different paths.

6. Acknowledgment

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7. References