Parameters and Steady State Characteristics Analysis of PMLSM

Abstract

Permanent magnet linear synchronous motor (PMLSM) has many advantages; it has been used in the industry. Electromagnetic parameters and steady state characteristics are the focus analysis of PMLSM. Based on a permanent magnet linear synchronous motor for horizontal motion system, according to the magnetic field unified equation, the thesis has given the formulas of the average air-gap flux density, harmonic analysis diagram of the air-gap magnetic field, the formulas of no-load leakage flux coefficient, no-load back electromotive force and the thrust in steady-state operation. The thrust-angle curve and structure parameters on the influence of the thrust are also analyzed, then the value of these parameters are computed according to the formula, the simulation curves to the parameters are given based on the finite element method. Two methods for the parameters are consistent with the experimental data, which also prove the correctness of the calculation methods and results.

Keywords. parameter, finite element method (FEM), steady state characteristics

1. Introduction

In many modern industry application, permanent magnet linear synchronous motors (PMLSM) have been an attractive choice because they have many advantages, and are applied to rope-less hoist system, high precision and high speed numerical control systems, ground transportations [1-2]. A lot of papers about PMLSM's theory analysis, design, simulation and experimental validation are studied [3-5]. The magnetic field analysis mainly focusing on which analysis methods to use is the foundation of electromagnetic parameter calculation, in order to get accurate magnetic field distribution, and then the characteristic parameters of PMLSM can be solved by magnetic circuit method according to the analysis results. Considering the complexity of the theoretical analysis, FEM is mainly concentrated on the magnetic field parameter's calculation [6-8]. In order to consider the tooth effect and end effect, at present there are some new methods to get more accurate magnetic field of the PMLSM [9, 10]. Papers adopt multi-layered model method to analyze every layer of the magnetic field and characteristic parameters in detail and lay the foundation for analysis theory of the PMLSM. Paper [11] adopts time-step FEM and field circuit coupling method to research load performance of the PMLSM. It is shown that load performance of the PMLSM in the heavy load condition is highly better than those in light load operation conditions. Thrust has an important effect on the steady-state characteristics of PMLSM, paper [12] has analyzed the flux linkage of PMLSM considering the end effect and harmonic effect, due to the end effect, the main harmonic of thrust is even harmonic, part of the harmonic component is proportional to q axis current, part of the harmonic component is proportional to the quadratic of q axis current. Paper [13] points out that thrust ripple, the cogging effect and the edge effect are the main reasons of thrust fluctuation, and the thrust ripple contains only 6 times fundamental wave frequency. The above papers on the analysis of the average air-gap flux density, no-load leakage flux coefficient, no-load emf, and thrust force with FEM in steady-state operation process are not much. Based on the paper [3-5], this paper adopts four-layer linear model theory on further analysis of motor parameters.
and steady state characteristics, has finished the calculation of the air-gap flux density, no-load leakage flux coefficient, no-load back emf, thrust. The thrust-angle curve and structure parameters on the influence of the thrust are also analyzed. The theory, simulation and experimental results show a good correlation and the rationality of calculation method.

2. Air-gap flux density analysis

2.1. Physical model of the magnetic Field

There are many commonly used analytical models of PMLSM’s air-gap magnetic field [3], such as magnetic circuit, multi-layered model method, direct integral method, equivalent magnetic network [14, 15], etc.. For the influence of tooth effect, there are three methods, equivalent coefficient of air gap, equivalent permeability in tooth-slot region and air gap permeance method. Equivalent coefficient of air gap method is attributed to the increase of the effective air gap. Air gap permeance method is to use Fourier analysis of air gap permeance, the magnetic field can be solved by the superposition of slotless magnetic field and air gap permeance, and it is very complex. Equivalent permeability in tooth-slot region method usually uses linear region with different permeability along the x and y direction instead of the actual tooth region [16], it is very convenient, so it can be used in this paper. The expressions of permeability along the x and y direction are respectively

\[
\mu_x = \frac{\mu_0 \mu_s}{1 + \frac{b_s}{t} (\mu - 1)}, \quad \mu_y = \mu_0 \left[ \frac{b_y}{t} + \mu_y (1 - \frac{b_y}{t}) \right]
\]  

(1)

Where, \( b_s \) is slot width, \( t \) is tooth pitch, \( \mu_0 \) is air-gap permeance.

Flux density amplitude can be attained by four-layer method, so this paper uses this method. Fig.1 shows the four-layer linear structure of PMLSM [3]. The model has four interface (1, 1', 2, 2', 3, 3', 4, 4'), Layer I and layer II which have different magnetic conductivity are tooth-slot layer or PM layer, region 1 and 4' are ferromagnetic materials layer whose magnetic field intensity is zero, they can be considered as the other two layer, so in essence, the model can be seen as two different magnetic conductivity \( \mu_{x1}, \mu_{y1} \) and \( \mu_{x2}, \mu_{y2} \) layer formed.

2.2. Air gap magnetic field analysis

The magnetic field unified equation can be derived as below [3-4]

\[
\frac{\partial E_x}{\partial y} = -j \omega \mu_0 H_z
\]

\[
\frac{\partial H_z}{\partial y} = j \frac{\pi^2}{\epsilon \omega \mu_0} E_x
\]

(2)

From eq. (2), eq. (3) is derived

\[
H_z = C_x e^{j\gamma} + C_y e^{-j\gamma}
\]

(3)
Where, \( y = \frac{\pi}{\tau} \sqrt{\frac{H_a}{\mu}} \)

From the linear model of Fig. 1, the boundary conditions are shown as below:

1) \( y = 0 : H_{x_1} = H_{x_2} = 0 \)
2) \( y = \eta : H_{x_2} - H_{x_2} = J_n, B_{y_2} = B_{y_2} \)
3) \( y = a : H_{x_3} = H_{x_4}, B_{y_3} = B_{y_4} \)
4) \( y = a + b : H_{x_4} = H_{x_4} = 0 \)

Where, \( J_n \) is the magnitude of the equivalent surface current density

From magnetic field unified equation (3) and boundary conditions (4), we can solve the magnetic field density at different region, and can calculate the steady-state operation characteristic parameters. So the air gap flux density expressions produced by the PM and armature winding along y axis respectively are

\[
B_{(y_2)} = -\mu_0 \frac{J_m \cdot s h y \phi_{h_m}}{T} \gamma_0^2 h_0 \delta
\]

\[
B_{(y_3)} = -\mu_0 \frac{J_m \cdot s h y \phi_{h_m}}{T} \gamma_0^2 h_0 \delta
\]

Where, \( J_m = \frac{4}{\tau} F_m \sin(\pi \alpha), J_m = \frac{\sqrt{2} m N_u K_{p_m}}{\tau p} \)

\( T = c h y h_0 \cdot \frac{\mu_0}{\sqrt{\mu_0 \mu}} s h y \cdot c h y_0 \phi_{h_0} (h_0 + \delta), T' = c h y_0 \phi_{h_0} (h_0 + \delta) + \frac{\sqrt{\mu_0 \mu}}{\mu_0} s h y_0 \phi_{h_0} (h_0 + \delta) \cdot c h y h_0 \)

\( B_{(y_2)} \) is the average air gap flux density produced by PM, \( B_{(y_3)} \) is the average air gap flux density produced by armature winding. \( J_m \) and \( J_m \) are the magnitude of the equivalent current density of PMs and armature winding respectively, \( F_m = H_0 \cdot h_0, \alpha = \frac{L_m}{\tau}, L_m \) is the length of the PM along x direction, \( \tau \) is pole pitch, \( m \) is phase number, \( N_u \) is the number of coil turns per phase, \( I \) is phase current, \( K_{p_m} \) is winding factor, \( p \) is pole pairs. \( h_0 \) is the height of armature winding, \( h_1 \) is slot depth, \( h_1 \) is the PM height, \( \delta \) is air gap length, \( \gamma_0 = \frac{\pi}{\tau} \), the parameters of motor are listed in Table 1.

<table>
<thead>
<tr>
<th>Phase number</th>
<th>3</th>
<th>Slot depth</th>
<th>28mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pitch</td>
<td>39mm</td>
<td>Conductors per slot</td>
<td>90</td>
</tr>
<tr>
<td>Core width</td>
<td>114mm</td>
<td>PM width</td>
<td>27mm</td>
</tr>
<tr>
<td>Core height</td>
<td>43mm</td>
<td>Coercivity</td>
<td>870KA/m</td>
</tr>
<tr>
<td>Slot number</td>
<td>18</td>
<td>PM height</td>
<td>7mm</td>
</tr>
<tr>
<td>Tooth pitch</td>
<td>13mm</td>
<td>Tooth width</td>
<td>5mm</td>
</tr>
</tbody>
</table>

Table 2. Air-gap flux density of BY fundamental component

<table>
<thead>
<tr>
<th>Fundamental wave of BY</th>
<th>PM only/T</th>
<th>Armature winding only/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four-layer method</td>
<td>0.691T</td>
<td>0.0872T</td>
</tr>
<tr>
<td>FEM</td>
<td>0.652T</td>
<td>0.0812T</td>
</tr>
</tbody>
</table>

We often use the magnitude of the flux density which can be attained from eq. (5) and (6) by four-layer method. We can also adopt FEM to attain the fundamental wave of the air-gap flux density through Fourier analysis. So the magnitude of the air-gap flux density along y direction by two methods shows in Table 2, it can be seen that the two results are very close.

The Fourier analysis results of BY produced by PM through FEM show in Fig. 2(a). It can be seen that the air gap flux density magnitude of fundamental wave is 0.652T, the 5th order harmonic
magnitude is 0.0878T, the other higher harmonics are very small. The Fourier analysis results of BY produced by armature winding through FEM show in Fig. 2(b). It can be seen that the air-gap flux density magnitude of fundamental wave is 0.0812T, the 5th order harmonic magnitude is 0.0263T, the other higher harmonics are also very small.

![Harmonics Distribution of PM](image1)

![Harmonics Distribution of Armature Winding](image2)

**Figure 2.** Harmonics Distribution of Air-Gap Flux Density BY

3. No-load leakage coefficient analysis

The magnetic field distribution of permanent magnet motor is very complex, it is related to the materials of the permanent magnet, different directions of magnetization, structure and many other factors. No-load flux leakage coefficient not only related to the utilization degree of permanent magnet material, but also has great influence on the demagnetization ability and the characteristic of permanent magnet motor. Accurate calculation of no-load flux leakage coefficient needs to be solved by 3D magnetic field of permanent magnet motor, but the amount of 3D calculation is very huge. Linear motor's air gap is big, so the no-load flux leakage coefficient is very large; the calculation of this paper has ignored the end effect, taking a pole pitch for solving region.

The no-load flux leakage coefficient $\sigma_0$ is the ratio of magnet flux $\Phi_m$ to air gap flux $\Phi_{d0}$, that is,

$$\sigma_0 = \frac{\Phi_m}{\Phi_{d0}}$$  (7)

Where, $\Phi_m = b_m B_A$, $b_m$ is the operating point without external excitations, the area of cross-section of the magnet and air gap are given as $A_m$ and $A_g$, respectively. $\Phi_{d0}$ is the average flux density of air gap produced by permanent magnet multiply by the area of cross-section of air gap, the following is obtained,

$$\Phi_{d0} = \frac{A_g}{T} \int_{y_0}^{y_0+\delta} B_{1/2}dy = \frac{A_g}{T} \int_{y_0}^{y_0+\delta} \mu_0 T \sqrt{\frac{\mu H}{\mu_0}} \left[ ch\gamma_0 (h_n + \delta - y) + \frac{\sqrt{\mu H}}{\mu_0} \right] dy$$

$$= \frac{A_g}{T} \mu_0 T \sqrt{\frac{\mu H}{\mu_0}} \left[ ch\gamma_0 \delta + \frac{\sqrt{\mu H}}{\mu_0} sh\gamma_0 \delta cth h_n - 1 \right]$$  (8)

From eq. (7) and (8), we have $\sigma_0 = \frac{b_m B_A}{\Phi_{d0}} = 1.2494$.

The no-load flux leakage coefficient can be calculated accurately by FEM, according to Stokes' theorem,

$$\Phi = \oint B \cdot ds = \int \nabla \times A \cdot ds = \int A \cdot dl$$  (9)
For 2D plane field, the absolute value of magnetic vector potential difference between two points is equal to the magnetic flux between these two points along Z axis of unit length. The solving region by FEM is shown in Fig. 3.

![Figure 3. Solving Region of the Motor](image)

So the no-load flux leakage coefficient solved by FEM is

\[
\sigma_0 = \frac{\Phi_{\text{m}}}{\Phi_{\text{no}}} = \left| \frac{A_3 - A_2}{A_4 - A_1} \right|
\]

(10)

Where, \( A_1, A_2, A_3, \) and \( A_4 \) are the magnetic vector potential of node 1, 2, 3, and 4 in Fig. 3 respectively. The magnetic vector potential of each node calculated by FEM in order is -0.014178, 0.006245, -0.012048, 0.004101. Table 3 shows the value of no-load flux leakage coefficient by magnetic circuit method and FEM.

<table>
<thead>
<tr>
<th>Method</th>
<th>Analysis method</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>No-load leakage coefficient</td>
<td>1.2494</td>
<td>1.2646</td>
</tr>
</tbody>
</table>

4. No-load back emf analysis

No-load back emf is produced by PM in the armature winding region, we have

\[
E_b = \frac{w}{\sqrt{2}} \left( N_s K_w \right) r h_b \left( \frac{2}{\pi} B_{(y3)\text{avg}} \right)
\]

(11)

Where, \( w \) is angular frequency, \( N_s K_w \) is the number turns in series per phase of armature winding, \( b_z \) is the length of PM along Z axis. \( B_{(y3)\text{avg}} \) is the average flux density produced by PM in the slot region, it can be attained from eq. (3) and (4).

\[
B_{(y3)\text{avg}} = \frac{\mu_\text{r} \mu_\text{i} J_{\text{w}} \sinh \gamma h_{\text{m}} \sinh \gamma h_{\text{y}}}{T' y_{\gamma} h_{\gamma}^2 y \sinh h_{\gamma}}
\]

(12)

Substitute eq.(12) into eq.(11), the no-load back emf is determined.

Table 4. The value of no-load emf

<table>
<thead>
<tr>
<th>Method</th>
<th>Analysis method</th>
<th>FEM</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average of no-load emf</td>
<td>9.128V</td>
<td>8.98V</td>
<td>8.5V</td>
</tr>
</tbody>
</table>

The no-load back emf calculated by FEM is to let the PM's speed in \( V_s = 0.312 \text{m/s} \), the armature winding with no electricity. Fig. 4 shows the curve of no-load emf by FEM, the curves of three phase emf are not symmetry because of the asymmetrical winding. The emf of ABC is 8.92927V, 8.8586V and 9.15606V respectively, the average no-load emf of three phases is 8.98V. Table 4 shows the results of no-load emf, it can be seen that the results calculated by analysis method, FEM and experimental data are very close.
5. Thrust analysis

5.1. Thrust calculation

Electromagnetic power is

$$P_n = 3E_s I_p = 3[(E_s U_s \cos \theta - E_r^2) r_s + X_r E_s U_s \sin \theta] / Z^2$$  \hspace{1cm} (13)

The thrust is

$$F_s = \frac{\pi P_n}{2\tau f} = \frac{P_n}{V_r}$$  \hspace{1cm} (14)

Where, $I_p$ is the active component of the current, $\theta$ is power angle, $r_s$ is armature resistance, $X_r$ is synchronous reactance, $Z$ is synchronous impedance, $Z = \sqrt{r_s^2 + X_r^2}$.

From (11)(13)(14) simultaneous equation, the normal thrust is solved as shown in Table 5.

The excitation source of solving region by FEM can be current source or voltage source, when voltage is 35V, speed is 0.312m/s, the simulation result of voltage source is shown in Fig. 5, because the structure is asymmetry, even if the voltage source is symmetrical, the current source is asymmetry, thrust ripple is very big and aperiodic, the average thrust is 253.932N. The amplitude of thrust ripple is about 150N, the main reason of it is detent force resulting from slotting effect and end effect [13-14].

According to the torque formula of motor theory, the thrust is related to the angle between stator and rotor magnetic potential, that is

$$T_s = C_n \cdot F_s \cdot F_r \cdot \sin \beta$$  \hspace{1cm} (15)

Where, $C_n$ is torque coefficient, $F_s$ is stator magnetic potential, $F_r$ is rotor magnetic potential, $\beta$ is the angle between stator and rotor magnetic potential.
5.2. The influence of structure parameters on the thrust

The structure parameters of PMLSM has an important impact on the performance of the motor, the optimization of different structural parameters can get the maximum thrust. Fig. 7(a) shows the maximum mean thrust characteristic with different lengths and thicknesses of permanent magnet; Fig. 7(b) shows the maximum mean thrust characteristic with different heights of armature winding. It can be seen that as the length of the permanent magnet increases, mean thrust grows nonlinearly, but the increasing rate decreases due to the core saturation when the thickness of permanent magnet is invariant. When the thickness of permanent magnet increases, mean thrust grows, the increasing rate also decreases; this is because the flux density grows maximum as the permanent magnet increases to a certain degree, then decrease as the air gap is large, leakage increase. When the height of armature winding increases, mean thrust also increases firstly then decreases, because the slot leakage flux gets bigger and bigger. Therefore, in the optimal design, thrust and motor characteristics must be considered synthetically, the structure parameters should be selected reasonably.

6. Conclusion

(1) The parameters of air gap flux density magnitude of fundamental wave, no-load leakage coefficient, no-load back emf and thrust are analyzed by analysis method and FEM. The steady-state characteristic curves of these parameters are given, the theory, simulation and experiment values have proved methods that are correct.
The expressions by magnetic circuit method have many equivalent parameters, which are not considered the end effect and magnetic saturation, but the results show that the expressions have high precision. The structure parameters and motor characteristics must be considered synthetically, the parameters should be selected reasonably.

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8. References