Life Prediction of Tantalum Capacitors Based on PSO-GM Model

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Abstract
Life prediction for Tantalum (Ta) capacitors is difficult by using conventional time-to-failure analysis method. Degradation analysis, which deals with parameters of performance degradation, is an efficient method to estimate reliability for highly reliable parts like Ta capacitors. Based on grey theory, the GM(1,2) model, a degradation analysis method for life prediction was proposed. Since the accuracy of GM(1,2) model was influenced deeply by parameters, weight parameter $\omega$ was optimized by particle swarm optimization method (PSO), and PSO-GM model was established. The practical test showed that the prediction errors were reduced by using the proposed PSO-GM model. It indicates that the proposed method is valid and accurate.

Key words: Grey Theory, Tantalum Capacitor, Performance Degradation, Life Prediction, Particle Swarm Optimization

1. Introduction
Life prediction for electronic components is necessary for reliability assembling. The reliability level of capacitors significantly affects the reliability and maintenance costs of those facilities. For military-grade tantalum (Ta) capacitors, traditional methods by using censored test or accelerate life test can predict life, but they need many destructive tests which cost much time and money, which limit their application. Since degradation data may provide considerably more information about reliability than censored failure-time data (especially with few or no failures). And degradation analysis can also save time and cost in testing. In recent years, the studies of performance degradation have attracted many interests and efforts because the degradation measurements contain fairly credible, accurate and useful information about product reliability. The grey model based on degradation proposed previously [1] can predict life of Ta capacitors (as well as many other high reliability and long life electronic components) with less time and cost. But its accuracy level still needs improving.

It is difficult to evaluate the reliability for Ta capacitors by traditional time-to-failure analysis method [2]. The performance of Ta capacitors deteriorates continuously over time by gradual degradation. The primary failure mode is soft degradation failure, and catastrophic failures rarely occur. Reliability prediction based on degradation analysis can be an efficient method to estimate reliability for highly reliable parts (like Ta capacitors) when observations of failures are rare [3]. Degradation analysis is based on probabilistic modeling of a failure mechanism degradation path and comparison of a projected distribution to a pre-defined failure threshold. It is a more advisable solution therefore.

Accelerated degradation test (ADT) is commonly used to predict lifetime of the components caused the degradation failure as time passed. It expedites product degradation by stressing the product beyond its normal use. To extrapolate the product’s reliability at use condition, ADT requires a known functional link relating the harsh testing environment to the usual use environment. Practitioners are often faced a great challenge to designate an explicit form of the stress-degradation relationship a priori in a accelerated degradation models [4].

As this testing system should be less impacted by outside environment, a highly-stable condition for data application is required. Grey theory method can be an advisable alternative in modeling and analyzing the degradation data. By constructing a sine function and adding proper buffer operators, an approximate grey model is formed. The GM(1,n) model [5] has been used into practice for its minimum requirements of data quantity in recent years. Weight parameter of the grey theory system has great influence of the accuracy of GM(1,n) model, but because of the nonlinear traits between the weight parameter and prediction errors, it is hard to identify its value. Particle swarm optimization (PSO), which is easy programmed, was used in this paper.
In this paper, a method for life prediction of Ta capacitors based on PSO-GM model was proposed. In Section 2, Ta capacitors’ structure and performances of degradation were introduced, as well as the failure modes. In Section 3, a GM(1,2) model was established firstly, then parameter \( \omega \) was optimized by PSO. Also the procedure of establishing a PSO-GM model was presented. A tantalum capacitor was taken as a case analysis in Section 4.

2. Ta Capacitor Performance Degradation Analysis

Ta capacitor (refers to solid Ta) utilizes solid manganese dioxide (MnO\(_2\)) as the counter electrode and exhibit excellent steady state reliability thanks to the inherent self-healing behavior. This self-healing process is an important factor in the steady state reliability characteristics of tantalum capacitors, which are referenced as having “no wear out mechanism”. One self-healing reaction is based on thermally inducing oxidization of the conductive MnO\(_2\) counter-electrode and converting into Mn\(_2\)O\(_3\) – a higher resistivity form of manganese oxide (shown as Fig 1).

![Fig 1] Structure of Ta capacitor

![Fig 2] Failure position of Ta capacitor

Ta capacitor is easy to be broken through by large instantaneous charge and discharge currents focusing on part of it. In addition, because of the compact process of its edges and corners, the holes in it are in different structure and size, which leads to high heat capacity and weak ability for distributing heat, gathering current together and causing overheat failures. Failure position is shown in Fig 2.

Leakage currents in chip tantalum capacitors are gradually increasing with time under highly accelerated life testing conditions at temperatures from 105\(^\circ\)C to 170\(^\circ\)C [6]. Leakage current \( I_r \) (wA) and capacity \( C \) (wF) are the two main parameters to characterize a Ta capacitor. During testing, define a Ta capacitor fail when any of the two parameters exceeds the prescribed value.

3. PSO-GM model

3.1. GM(1,2) model

Grey system theory is very suitable for computation relationship between series that have uncertain life distributions and small samples. Different from the GM(1,1) model ever widely used[7,8], GM(1,2) utilizes double-parameter to reach a higher accuracy, establishing a first-order linear dynamic model with two sequences. The model is simulated by differential equations from adding data.

First, the observations series for the selected sensitive parameters of Ta capacitors is assumed to be \( X(0) \)

\[ \{x_i^{(0)}(k)\} \ (i=1, 2; \ k=1, 2, 3, \ldots) \] (1)

where \( i \) is the number of selected sensitive parameters, \( k \) is the serial number, \( X(1) \) is the accumulated generating series for the corresponding sensitive parameters, that is

\[ x_i^{(1)}(k) = \sum_{j=1}^{k} x_i^{(0)}(j) \] (2)

where \( k = 1, 2, \ldots, m \).

If the original series of the selected sensitive parameters and their accumulated generating sequence are expressed as matrix, then the \( k \) line of the matrix can be expressed as

\[
\begin{align*}
X^{(0)}(k) &= [x_1^{(0)}(k), x_2^{(0)}(k)]^T \\
X^{(1)}(k) &= [x_1^{(1)}(k), x_2^{(1)}(k)]^T
\end{align*}
\] (3)

A first-order ordinary differential equations of the accumulated generating series can be obtained as
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\[ \begin{align*}
\frac{dx_1^{(1)}(t)}{dt} &= a_{11}x_1^{(1)}(t) + a_{12}x_2^{(1)}(t) + b_1 \\
\frac{dx_2^{(1)}(t)}{dt} &= a_{21}x_1^{(1)}(t) + a_{22}x_2^{(1)}(t) + b_2
\end{align*} \]  
(4)

Let \( A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \), \( B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \). Eq.(4) can be written as

\[ \frac{dx^{(1)}(t)}{dt} = AX^{(1)}(t) + B \]  
(5)

Then the general difference scheme of the proposed model can be obtained as

\[ X^{(1)}(t + \Delta t) - X^{(1)}(t) = A \left[ \omega X^{(1)}(t) + (1 - \omega)X^{(1)}(t + \Delta t) \right] + B \]  
(6)

where \( \omega \) is the weight parameter, \( 0 \leq \omega \leq 1 \).

If \( a_i = \begin{bmatrix} a_{i1} & a_{i2} & b_i \end{bmatrix} \) \((i=1,2,\ldots)\), \( a_i \) can be obtained by the least squares method (LSM) as

\[ \hat{a}_i = \left[ \hat{a}_{i1} \hat{a}_{i2} \hat{b}_i \right] = (L^T L)^{-1} L^T Y_i \]  
(7)

where

\[ L = \begin{bmatrix} \omega x_i^{(1)}(1) + (1 - \omega)x_i^{(1)}(2) & \omega x_i^{(1)}(2) + (1 - \omega)x_i^{(1)}(3) & \cdots & \omega x_i^{(1)}(m - 1) + (1 - \omega)x_i^{(1)}(m) \\
\omega x_i^{(1)}(2) + (1 - \omega)x_i^{(1)}(3) & \omega x_i^{(1)}(3) + (1 - \omega)x_i^{(1)}(4) & \cdots & \omega x_i^{(1)}(m) + (1 - \omega)x_i^{(1)}(m) \\
\vdots & \vdots & \ddots & \vdots \\
\omega x_i^{(1)}(m) + (1 - \omega)x_i^{(1)}(m) & \omega x_i^{(1)}(m) + (1 - \omega)x_i^{(1)}(m) & \cdots & \omega x_i^{(1)}(m) + (1 - \omega)x_i^{(1)}(m) \end{bmatrix} \]

\[ Y_i = \begin{bmatrix} x_i^{(0)}(2) \\ x_i^{(0)}(3) \\ \vdots \\ x_i^{(0)}(m) \end{bmatrix} \]

Then parameters \( A \) and \( B \) can be expressed as

\[ \hat{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{bmatrix} \]

\[ \hat{B} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \end{bmatrix} \]  
(9)

The \( l \) component of \( \{x_l^{(1)}(k)\} \) is \( \{x_l^{(1)}(l)\} \), which is assumed as the initial condition. Then continuous-time response for Eq.(5) can be obtained as

\[ X^{(1)}(t) = e^{At}X^{(1)}(l) + A^{-1}(e^{At} - I)B \]  
(10)

where \( e^{At} = I + At + \frac{A^2}{2!}t^2 + \cdots = I + \sum_{k=1}^{\infty} \frac{A^k}{k!} t^k \).

Then the double-parameter model can be written as

\[ \bar{X}^{(1)}(k) = e^{\bar{A}(k-l)}X^{(1)}(l) + \bar{A}^{-1}(e^{\bar{A}(k-l)} - I)\bar{B} \]  
(11)

where \( k = 1,2,\ldots,m \), \( 1 \leq l \leq m \). Then

\[ \bar{X}^{(0)}(k) = \bar{X}^{(1)}(k) - \bar{X}^{(1)}(k-1) \]  
(12)

3.2. Optimization of \( \omega \) by PSO

In GM (1,2) model, \( \omega \) values 0.5 by experience. According to the simulation result (shown in Fig 3), the predicted curve changed vividly by different values of \( \omega \). To improve the prediction accuracy, PSO was proposed to optimize it. PSO is an adaptive algorithm based on a social-psychological metaphor [9]. This process is iterated a set number of times until a minimum error is achieved. The governing equations are as follows

\[ \begin{align*}
\frac{dx_1^{(1)}(t)}{dt} &= a_{11}x_1^{(1)}(t) + a_{12}x_2^{(1)}(t) + b_1 \\
\frac{dx_2^{(1)}(t)}{dt} &= a_{21}x_1^{(1)}(t) + a_{22}x_2^{(1)}(t) + b_2
\end{align*} \]
where $\lambda$ is inertia weight, $c_1$, $c_2$ are acceleration coefficients, $r_1$, $r_2$ are random number between 0 and 1, $p_{id}$ is the best position of each particle and $p_{gd}$ is the global best particle.

\[
\begin{align*}
v_{id} &= \lambda v_{id} + c_1 r_1 (p_{id} - x_{id}) + c_2 r_2 (p_{gd} - x_{id}) \\
x_{id} &= x_{id} + v_{id} 
\end{align*}
\]  

(13)

Fig 3  Simulation curves under different $\omega$

The process can be described as the following five steps.

Step 1) Initialization. Take $M$ as particle size, create initial group $(\omega_{0i})_i^M$. Initialize inertia weight $\lambda$, acceleration coefficient $c_1,c_2$, the maximal generation(GEN), the initial position and velocity of every particle.

Step 2) Fitness function. Take min (abs(MRE)) as fitness function, viz:

\[
\min f(\omega) = \min \left[ \frac{1}{n} \sum_{k=1}^{n} \left( x^{(0)}(k) - x^{(0)}(\omega k) \right) \cdot 100 \right] 
\]

Step3) Operation. If $f(\omega_i) < p_{id}$, then $p_{id} = f(\omega_i)$.

Step4) Updating calculate the new velocity and position of every particle according to Eq.(13).

Step5) Stop rule. When a minimum error is achieved or iterative number attains GEN, stop searching and output searching result, otherwise return to step 2 and go on searching.

The flowchart of PSO-GM is made up of the GM(1,2) and PSO, as shown in Fig 4.
4. Application

4.1. Data collecting

The experiment chose Ta capacitor marked as CAK45 16V68uF for example. It was conducted at three different stress levels 85°C, 120°C, and 145°C. In this paper, leakage current and capacitance of CAK45 16V68uF were noted. Data obtained from stress levels 85°C and 120°C was used to estimate the model parameters. Data obtained from stress levels 145°C was used to validate the model.

At each stress levels we conduct one accelerated life testing experiment with 5 samples for testing. In each test a designed-circuit-board that contains 10 randomly chosen Ta capacitors is placed in a temperature chamber where the temperature and voltage in the circuit are held constant. The leakage current and capacitance of the devices were measured at room temperature every 192 hours and 72 hours, respectively. Table 1 and table 2 show the accelerated degradation testing data of leakage current and capacitance over time at each temperature stress level, respectively.

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Sample</th>
<th>Leakage current degradation value (μA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 (85°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#1</td>
<td>0.55</td>
<td>0.56 0.58 0.64 0.65 0.69</td>
</tr>
<tr>
<td>#2</td>
<td>0.51</td>
<td>0.55 0.57 0.59 0.64 0.68</td>
</tr>
<tr>
<td>#3</td>
<td>0.49</td>
<td>0.52 0.55 0.6  0.63 0.67</td>
</tr>
<tr>
<td>#4</td>
<td>0.51</td>
<td>0.54 0.56 0.63 0.65 0.69</td>
</tr>
<tr>
<td>#5</td>
<td>0.48</td>
<td>0.53 0.55 0.57 0.62 0.65</td>
</tr>
<tr>
<td>S2 (120°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#6</td>
<td>0.48</td>
<td>0.63 0.79 0.95 1.09 1.24</td>
</tr>
<tr>
<td>#7</td>
<td>0.49</td>
<td>0.65 0.8  0.96 1.11 1.27</td>
</tr>
<tr>
<td>#8</td>
<td>0.55</td>
<td>0.71 0.85 1  1.16 1.3</td>
</tr>
<tr>
<td>#9</td>
<td>0.52</td>
<td>0.67 0.83 0.97 1.12 1.26</td>
</tr>
<tr>
<td>#10</td>
<td>0.53</td>
<td>0.68 0.81 0.95 1.11 1.25</td>
</tr>
<tr>
<td>S3 (145°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>#11</td>
<td>0.49</td>
<td>0.99 1.5  2.01 2.49 2.98</td>
</tr>
<tr>
<td>#12</td>
<td>0.51</td>
<td>1.02 1.51 2.02 2.51 2.99</td>
</tr>
<tr>
<td>#13</td>
<td>0.49</td>
<td>0.98 1.49 2  2.48 2.98</td>
</tr>
<tr>
<td>#14</td>
<td>0.53</td>
<td>1.04 1.53 2.04 2.54 3.03</td>
</tr>
<tr>
<td>#15</td>
<td>0.48</td>
<td>0.97 1.46 1.98 2.47 2.98</td>
</tr>
</tbody>
</table>
### Table 2 Capacitance degradation data under ADT

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Sample</th>
<th>Capacitance degradation value (μF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 (85 ℃)</td>
<td>#1</td>
<td>68.013 68.114 68.209 68.293 68.381 68.496</td>
</tr>
<tr>
<td></td>
<td>#2</td>
<td>68.032 68.143 68.228 68.316 68.392 68.521</td>
</tr>
<tr>
<td></td>
<td>#3</td>
<td>68.003 68.097 68.172 68.258 68.331 68.413</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>67.985 68.085 68.161 68.231 68.317 68.408</td>
</tr>
<tr>
<td></td>
<td>#5</td>
<td>67.932 68.079 68.159 68.227 68.313 68.398</td>
</tr>
<tr>
<td>S2 (120 ℃)</td>
<td>#6</td>
<td>67.993 68.244 68.489 68.713 68.921 69.156</td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>68.005 68.249 68.508 68.726 68.942 69.201</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>68.012 68.257 68.502 68.758 69.031 69.233</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>67.988 68.245 68.501 68.731 68.987 69.198</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>68.032 68.279 68.52 68.767 69.013 69.258</td>
</tr>
<tr>
<td>S3 (145 ℃)</td>
<td>#11</td>
<td>67.989 68.483 68.995 69.509 69.988 70.497</td>
</tr>
<tr>
<td></td>
<td>#12</td>
<td>68.023 68.513 69.004 69.521 70.055 70.529</td>
</tr>
<tr>
<td></td>
<td>#13</td>
<td>67.997 68.449 68.952 69.495 70.002 70.487</td>
</tr>
<tr>
<td></td>
<td>#14</td>
<td>68.011 68.477 68.956 69.476 70.073 70.507</td>
</tr>
<tr>
<td></td>
<td>#15</td>
<td>68.006 68.483 68.985 69.487 70.083 70.511</td>
</tr>
</tbody>
</table>

### 4.2. Model analyzing

According to steps shown in Section 3, a PSO-GM model was established. A comparison in error value between GM(1,2) model and PSO-GM model was made, shown in Table 3, where \( \sum_{k=1}^{6} \omega(k) \) was the sum of a sample’s relative errors between original and prediction data.

\[
\sum_{i=1}^{15} \sum_{k=1}^{6} \omega_i(k) \quad \text{was the sum of} \quad \sum_{k=1}^{6} \omega(k) \text{from all samples.}
\]

From the data shown in Table 3, it can be seen that the fitting accuracy of degradation data by GM(1,2) model is 97.45%, while by PSO-GM model is 98.75%. The accuracy of PSO-GM model has increased 1.32% than that of GM(1,2) model, showing the efficiency of PSO-GM model as proposed.

### Table 3 Comparison of GM(1,2) model and PSO-GM model

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Sample</th>
<th>GM(1,2) model</th>
<th>PSO-GM model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \sum_{i=1}^{6} \omega_i(k) )</td>
<td>( \sum_{k=1}^{6} \omega(k) )</td>
</tr>
<tr>
<td>S1 (85 ℃)</td>
<td>#1</td>
<td>0.324501</td>
<td>0.057108</td>
</tr>
<tr>
<td></td>
<td>#2</td>
<td>0.316505</td>
<td>0.054963</td>
</tr>
<tr>
<td></td>
<td>#3</td>
<td>0.365836</td>
<td>0.027577</td>
</tr>
<tr>
<td></td>
<td>#4</td>
<td>0.334275</td>
<td>0.064842</td>
</tr>
<tr>
<td></td>
<td>#5</td>
<td>0.32604</td>
<td>0.030694</td>
</tr>
<tr>
<td>S2 (120 ℃)</td>
<td>#6</td>
<td>0.281191</td>
<td>0.13596</td>
</tr>
<tr>
<td></td>
<td>#7</td>
<td>0.249616</td>
<td>0.103523</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>0.238513</td>
<td>0.086258</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>0.367907</td>
<td>0.121435</td>
</tr>
<tr>
<td></td>
<td>#10</td>
<td>0.237086</td>
<td>0.069855</td>
</tr>
<tr>
<td>S3 (145 ℃)</td>
<td>#11</td>
<td>0.311588</td>
<td>0.311679</td>
</tr>
<tr>
<td></td>
<td>#12</td>
<td>0.34489</td>
<td>0.287161</td>
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<tr>
<td></td>
<td>#13</td>
<td>0.358782</td>
<td>0.311714</td>
</tr>
<tr>
<td></td>
<td>#14</td>
<td>0.219025</td>
<td>0.277206</td>
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<tr>
<td></td>
<td>#15</td>
<td>0.305656</td>
<td>0.296366</td>
</tr>
</tbody>
</table>

### 4.3. Life prediction

According to Arrhenius Model [10, 11], a model related to temperature has been established

\[
\ln \xi = a + b / T \tag{14}
\]

Where \( a \) and \( b \) are the coefficients to be determined. \( T \) is the temperature and \( \xi \) is life trait.

By using two groups of pseudo-life [12] and temperature data, the coefficients \( a, b \) in Eq.(14) can be
obtained as $a = -2.8872$, $b = 4349.5$. When working temperature of the components is $35 \, ^\circ C$, the pseudo-life can be obtained as

$$
\mu_0 = \exp(-2.8872 + \frac{4349.5}{35 + 273}) = 76462 \text{(hours)}
$$

That is, the predicted life of Ta capacitors marked as CAK45 16V68uF is approximately 8.73years, which is corresponding to 8-10 years provided by manufacturer.

5. Conclusion

A life prediction method based on PSO-GM model with predicted error improvement for Ta capacitors was proposed. By analyzing Ta capacitors' structure characters, the degradation performance of Ta capacitor was introduced first. A GM(1,2) model was applied to solve the problem of small samples. Because of the weakness of identifying weight parameter $\omega$ in GM(1,2) model, PSO was applied to optimized $\omega$. The PSO-GM model was validated experimentally by conducting an accelerated testing on CAK45 16V68uF capacitors, at three different stress levels $85 \, ^\circ C$, $120 \, ^\circ C$, and $145 \, ^\circ C$. Results have shown that the prediction accuracy has been further improved compared to conventional GM(1,2) model.

6. Acknowledgment

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7. References

[8]. Wei Xiong, Aiping Yang, Wenzhan Dai, "Improvement of background value and its application in non-equidistance GM(1,1) modeling", in Proceedings of 8th World Congress on Intelligent Control and Automation(WCICA), pp. 5919-5923, 2010.
[13]. HUANG Jiao-ying Huang, GAO Cheng, CUI Wei and MEI Liang, “Lifetime prediction for

