Nonlinear Model of Permanent-Magnet Synchronous Motors

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Abstract

This paper develops a comprehensive nonlinear model of permanent-magnet synchronous motors (PMSM) to be used for simulation and control models. The model is described in a new expression to estimate the torque with low volume offline data. The offline data is constructed by magnetic flux and cogging torque and inductance. When the magnetic saturation, the magnetic flux and inductance is changed with the phase current and the position of rotor. In order to establish the nonlinear relation among magnetic flux, inductance and phase current, and the position of rotor, least squares support vector machine (LS-SVM) is used. The sample of LS-SVM is obtained from static finite element analyses (FEA) simulations. The experimental results show that the proposed model is more accurate than the conventional model.

Keywords: Nonlinear Model, Least Squares Support Vector Machine, Finite Element Analyses

1. Introduction

The permanent-magnet synchronous motors (PMSM) are widely used in various electrical devices due to their high torque-to-current, power-to-weight ratios and their high efficiency. The conventional model of PMSM is $d-q$ axis frame, and this model assumes a constant value of induced electromotive (EMF) due to permanent-magnets. The conventional model present method of calculating the reluctance torque is performed under the assumption that all load current and the magnetic flux flow only along the two-axis [1-3]. But due to the saturation in the rotor and stator cores the $d-q$ axis quantities are no longer independent of each other. So the conventional model is difficult to accurately calculate the magnet torque generated from direct- and quadrature-axis flux linkage due to permanent-magnets, it is difficult to calculate the inductance value, because the flux linkage due to permanent-magnets and inductance exist a nonlinear relation with current value and rotor position.

This paper presents a nonlinear model of PMSM, which uses the LS-SVM to predict flux linkage due to permanent-magnets and inductance. The sample of LS-SVM is obtained from static finite element analyses (FEA) simulations. The cogging torque is also taken account into the model, the proposed model is build based on the Simulink model. By comparing with the simulation result and experimental test, the proposed model is able to provide better accuracy.

2. Conventional model of PMSM

In the conventional two-axis machine model, the electrical subsystem of conventional two-axis PMSM model is described by voltage equations [4,5]:

\[ \]
The flux linkage equations of conventional two-axis PMSM model is described as:

$$\psi_d = L_d i_d + L_{mf} i_f$$
$$\psi_q = L_q i_q$$

(2)

The torque equation of conventional two-axis PMSM model is described as:

$$T_{em} = p (\psi_d i_q - \psi_q i_d) = p [L_{mf} i_f + (L_d - L_q) i_d i_q]$$

(3)

In the equations (1), (2) and (3), $u_d$ and $u_q$ are direct and quadrature-axis voltage, $i_d$ and $i_q$ are direct and quadrature-axis current, $R$ is phase resistance of armature coils, $\psi_d$ and $\psi_q$ are direct- and quadrature-axis flux linkage, $L_d$ and $L_q$ are inductance of direct- and quadrature-axis, $L_{mf}$ is permanent-magnet equivalent inductance, $i_f$ is permanent-magnet equivalent current, respectively.

3. Proposed nonlinear model of PMSM

3.1. Torque calculated by magnetic co-energy

In electric motors, the output torque is expressed as follow [5-7]:

$$T_e = - \frac{\partial W(i, \theta)}{\partial \theta}$$

(4)

Where $W$ is magnetic co-energy, $i$ is armature current, respectively. The $W$ can be expressed as follow:

$$W = W_r + W_m + W_m$$

(5)

Where $W_r$ is the magnetic co-energy due to only the armature current, $W_m$ is the magnetic co-energy due to interaction of armature and the magnet, $W_m$ is the magnetic co-energy due only the magnet. Through analyzing the three energy terms by energy theory, they can expressed as follow [8]:

$$\frac{dW_r}{d\theta} = \frac{1}{2} L \frac{d}{d\theta} i^2$$

(6)

$$\frac{dW_m}{d\theta} = pi \frac{d\psi}{d\theta}$$

(7)

$$\frac{dW_m}{d\theta} = T_{cogging}$$

(8)
In the formula (6), (7) and (8), \( p \) is the number of pole pairs, \( L \) is inductance, \( \psi \) is magnetic flux, \( T_{\text{cogging}} \) is cogging torque. Then the torque equation can expressed as follow[13-15]:

\[
T = \frac{1}{2} p L \left( \frac{di}{d\theta} \right)^2 + p i \frac{d\psi}{d\theta} + T_{\text{cogging}}
\]

The voltage, flux linkage equation are given in equation (10) and (11).

\[
V = R i + \frac{d\psi}{dt}
\]

\[
\psi = L i + \psi_{\text{m}}
\]

### 3.2. Inductance and magnetic flux linkage calculate use LS_SVM

#### 3.2.1. Theory of LS-SVM

The main objective of regression estimation is to approximate a function \( f(x) \) from a given set of samples \((x_i, y_i)\). SVM approximates the function as follows[9]:

\[
f(x) = \omega^T \phi(x) + b
\]

\( \phi(x) \) denotes a set of non-linear transformations from the low dimensional space to the high dimensional feature space. SVM regression is formulated as minimization of the following function:

\[
\min_{\omega} \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^{n} (\xi_i + \xi_i^*)
\]

Subject to: \( y_i - \omega^T \phi(x_i) - b \leq \xi_i + \xi_i^* \) and \( \omega^T \phi(x_i) + b - y_i \leq \xi_i + \xi_i^* \), where \( \xi \) is called tube size, \( \xi_i \) and \( \xi_i^* \), \( i \) are the slack variables, \( C \) is the regularization parameter. By introducing Lagrange multipliers, decision function (12) takes the following form:

\[
f(x) = \sum_{i=1}^{n} (a_i - a_i^*) \phi(x_i) + b
\]

Where \( \phi(x, \xi) \) is kernel function. \( a_i \) and \( a_i^* \) are Lagrange multipliers which are obtained by maximizing the dual form of the function (13). The dual form is as follows:

\[
\max \mathcal{W}(\omega) = \sum_{i=1}^{n} y_i (a_i - a_i^*) - \frac{1}{2} \sum_{i=1}^{n} (a_i - a_i^*) (a_i - a_i^*) \phi(x_i, \xi)
\]

Subject to: \( \sum_{i=1}^{n} (a_i - a_i^*) = 0 \), \( 0 \leq a_i, a_i^* \leq C \), \( i = 1, 2, \ldots, l \).

LS-SVM is trained by the following equation:

\[
\min J = \frac{1}{2} \omega^T \omega + \frac{1}{2} \sum_{i=1}^{n} \xi_i^2
\]

Subject to the equality constraints: \( y_i = \omega^T \phi(x_i) + b + \xi_i \).

Then the optimization problem is expressed as the following linear equation:
\[
\begin{bmatrix}
0 & I_v \\
I_v & K + y^{-1}I
\end{bmatrix}
\begin{bmatrix}
K' \\
y
\end{bmatrix}
= \begin{bmatrix}
0 \\
y
\end{bmatrix}
\]

Where \( y = [y_1, \ldots, y_N] \), \( I_v = \{1, \ldots, I_v\} \), \( a = [a_1, \ldots, a_I] \), \( K = \{k_i\} \), then we gain the nonlinear model of LS-SVM:

\[
f(x) = \sum_{i=1}^{N} (a_i - \alpha_i)K(x, x_i) + b (17)
\]

### 3.2.2. LS-SVM model for calculate inductance and magnetic flux linkage

Theoretically, the inductance and magnetic flux linkage of PMSM is not only the function of rotor position but also the winding currents, then we use LS-SVM express the nonlinear relation between them. The LS-SVM model for calculating the inductance and magnetic flux linkage of PMSM can be expressed as follow:

\[
L(x) = \sum_{i=1}^{N} (a_i - \alpha_i)K(x, x_i) + b (18)
\]

\[
\psi(x) = \sum_{i=1}^{N} (a_i - \alpha_i)K(x, x_i) + b (19)
\]

Where \( x \) represents the winding current \( i \) and the rotor position \( \theta \). The sample of LS-SVM is obtained from static finite element analyses (FEA) simulations.

**Figure 1.** LS-SVM model for inductance

**Figure 2.** LS-SVM model for PM flux linkage

### 4. Simulation use nonlinear model of PMSM

#### 4.1. Cogging torque and sample for LS-SVM model use FEA

In the nonlinear model of PMSM, the cogging torque data and the sample for LS-SVM model of PMSM are obtained use the method of FEA. We firstly build the FEA model of PMSM. The mathematic formula for magnetic vector potential and boundary condition express as equation (20), FEA model of PMSM show as Figure 3. Its parameters list in Table 1. We use ansoft Maxwell to solve the FEA model of PMSM by parameterization method. We can obtain the data which is needed for nonlinear model of PMSM[10,11,12].

\[
\nabla^2 A_x = \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} = -\mu J_x, A_x = 0, A_y = \pm A_y \bigg|_{n}, \frac{1}{\mu} \frac{\partial A_x}{\partial n} \bigg|_{n} = \frac{1}{\mu} \frac{\partial A_y}{\partial n} \bigg|_{n} = J_n (20)
\]
4.2. Simulink block of nonlinear model PMSM

The Simulink block of nonlinear model PMSM is build based on formula (4)-(19), which contain mainly 3 blocks as Figure 4-6.

![Simulink block for voltage calculation of nonlinear model PMSM](image)

**Figure 4.** The simulink block for voltage calculation of nonlinear model PMSM
4.3. Analyze the result of simulation and experiment

Firstly, we use the method of FEA, and cogging torque data and 50 samples for LS-SVM model of PMSM are obtained. The parameters of LS-SVM model as $\gamma = 3000$, $\sigma = 0.1$. Then the data which is input simulink blocks are obtained. The simulation time is 46.5s, the result of simulation is shown as Figure 7-8. In order to validate the accuracy of the proposed model, we put up the experiment platform, and conventional simulation model of PMSM is build too. The result of experiment and simulation are shown as Figure 9-10, the average amplitude of torque ripples of proposed model is 0.23N\(^*\)m, the average amplitude of torque ripples of conventional
model is 0.12N*m, the average amplitude of torque ripples of experiment is 0.26N*mm, So the accuracy of the proposed model is much higher than that of conventional model.

5. Conclusions

We have created a nonlinear model of PMSM that accounts for several nonlinear factors, which affects the operation of PMSM. In order to establish the nonlinear relation among magnetic flux linkage, inductance and phase current, and the position of rotor, the least squares support vector machine (LS-SVM) is used. The sample of LS-SVM is obtained from static finite element analyses (FEA) simulations. The simulation results show that the proposed model is more accurate than the conventional model. It can be used in simulation of PMSM control frames.

6. References


