Aircraft Financial Leasing Project Risk Evaluation Using AHP and Grey Comprehensive Algorithm

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Abstract

Aircraft financial leasing project risk is primarily qualitative studies, and for the airline, quantitative risk assessment is more advantageous to make decision. Through literature research and investigation, a grey comprehensive evaluation index system of Aircraft financial leasing is built, and it uses AHP to calculate the weights of indicators with consistency test. Finally a grey comprehensive assessment model of risk for aircraft financing leasing is proposed, and a case adopting this method is cited in this paper. The proposed method is effective.

Keywords: Aircraft Financial Leasing, Risk Assessment, Grey Comprehensive Assessment

1. Introduction

Aircraft leasing as an important way of introducing aircraft for local airlines began in 1960, when United Airlines leased a civilian aircraft by leveraged lease [1][2]. Since then, aircraft leasing has become one of the most primary way of introducing aircraft among global airlines. Civilian aircraft leasing in China began in 1980. During the past 30 years, the airlines have introduced a large number of aircrafts by leasing and now the number still is growing rapidly. According to the statistics, there were about 1405 aircrafts in Civil Aviation industry of China by the end of 2009. Among them, 801 aircrafts are introduced by finance leases which is 57% of the number counted. In recent years, the proportion of aircraft leasing has further increased due to fuel price rising, limited free funds and other factors.

There are many advantages of aircraft leasing compared with aircraft buying, including saving aircraft delivery time, reducing aircraft delivery period, earlier replacement of the old models, rapid expansion of the fleet, and thereby more flexibility of the airline operators [3][4] [5][6]. But there are also some disadvantages, such as complex leasing transactions, particular restrictions of use without ownership, troublesome surrender of tenancy. Above all, the biggest drawback of aircraft leasing is the risk of financial lease.

Chinese scholars mainly focused on the qualitative analysis on the risk of aircraft financial leasing. The main content of those theoretical studies are the defining of aircraft leasing risk and the identification of specific risks.

Liu Dedong et al. [7] developed a hierarchy selection model with 5 ecological bank-protection schemes and 7 decision criteria, using the analytic hierarchy process (AHP) for optimal ecological bank-protection method of channel in plain area. For a given plain-area channel, the model results indicate that the criteria of bank-protection including ecological restoration capacity, support capacity and service life are most important in decision-making process.

Du Yan Ping et al. [8] proposed fuzzy analytic hierarchy process (FAHP) combined qualitative and quantitative analysis to evaluate the reason of the active train plan. Firstly, an evaluation index system for high-speed railway train plan on the special operation conditions is built, and the indexes are made dimensionless. Then the relative weights of evaluation indexes are determined by analytic hierarchy process, and high-speed railway train plans on the special operation conditions are evaluated by comprehensive fuzzy evaluation method. Finally, Beijing-Shanghai high-speed rail is taken as example. The special operation condition is simulated and the corresponding train plans are created and evaluated.

On the review of the above studies, a comprehensive evaluation model on aircraft financial leasing risk is proposed for the airlines with a method based on AHP and gray comprehensive evaluation method and attempt to provide guide for domestic lease and to improve risk management level.
The rest of the paper is organized as follows. Section 2 is the description of establishing the gray comprehensive evaluation model. Section 3 focuses on application of the model. Finally, we end this paper with a conclusion and the future work.

2. Establish the gray comprehensive evaluation model

There are seven steps to establish the gray comprehensive evaluation model.

Step 1. The index system

Following the basic AHP procedures, the first thing is to establish evaluation index system on the risk of financial leasing. Based on the literature and field investigation data, and following the principle of comprehensiveness, horizontal comparability, the index system is established with six types of risks, including economic environment, financial risks, operational risks, security risks, technical development risks, residual value risk. The index system can be divided into the three levels as follows: target level, intermediate level, indicator level.

Target level: The main aim of this article is to evaluate the risk of the aircraft financial leasing risks, so the target level is marked as A.

The intermediate level: The risks are divided into six types: economic environment, financial risks, operational risks, security risks, technical development risks, and residual value risk. They are marked as B1, B2, B3, B4, B5, and B6. And the weight of each index is marked as \( W = (W_1, W_2, W_3, W_4, W_5, W_6) \).

The indicator level: The indicator level is the specification of the intermediate level. Economic environment B1 is specified into 3 indicators: C1 is industry prospect, C2 is industry competition and C3 is environmental protection. The financial risks B2 are specified into 3 indicators: the change of tax law risk (C4), the interest rate risk (C5), the foreign exchange risk (C6). Operational risks (B3) are specified into 2 indicators: the cash flow risk (C7) and the price risk of aviation gasoline and aviation supplies (C8). Security risks are specified into 3 indicators: manipulate error or malicious damage (C9), aircraft mechanical failure (C10), natural and force majeure damage (C11); risk of technical development (B5) are specified into 2 indicators, risk of the technical maturity (C12) and risk of lack of technical staff (C13); Risk of residual value (B6) are specified into 3 indicators: risk of overestimated residual value (C14), risk of decline in demand of old aircraft market (C15) and risk of residual value of aging aircrafts (C16); Detailed index system is in the Figure 1 below. The index weights are marked as \( W_i = (w_{i1}, w_{i2}, w_{i3}, ..., w_{i6}) \).

Step 2. The establishment of the judgment matrix and the confirm of the weight

After construction of the multiple-level analyzing structural module, the judgment matrix is constructed by Pair wise comparisons. In judgment matrix, the judge of the degree of the relative importance of elements in one level. Then though resolving the largest characteristic value \( \lambda_{\text{max}} \) and the corresponding eigenvectors in judgment matrix, we can get the relative weight of those elements. This is also the basic principles of the AHP Analytic Hierarchy Process. In a complex systematic research, constructing a judgment matrix and introducing reasonable metrics is the way of measuring relative importance among elements for some immeasurable factors.

Following the way of the value assignment in judgment matrix (table 1), examine the indicators one by one in the same level with comparative method and then get the judgment matrix.
Table 1. The meaning of proportion scale value assignment

<table>
<thead>
<tr>
<th>scale value</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>two factors of the same significance</td>
</tr>
<tr>
<td>3</td>
<td>two factors, the first one of slight significance</td>
</tr>
<tr>
<td>5</td>
<td>two factors, the first one of obvious significance</td>
</tr>
<tr>
<td>7</td>
<td>two factors, the first one of much more significance</td>
</tr>
<tr>
<td>9</td>
<td>two factors, the first one of extremely more significance</td>
</tr>
</tbody>
</table>

For a better description of the indicators and the reasonable value assignment on constructing judgment matrix, the rules are as follows: the value between scale values can also be used to describing significance. If its significance is between “same significance” and “slight significance”, the scale value should be 2. If its significance is between “slight significance e” and “obvious significance”, the scale value should be 4, and so on. In the meanwhile, the value in line i and row j means the relative importance of element in line i to element in row j. Therefore, the value in diagonal is 1. And we also make a rule that if relative weight of element in line i to j in row j is n, then the relative weight of element j to i is 1/n. then we get the judgment matrix as follows (see table 2 and table 3):

Table 2. A-B judgment matrix

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B₁</th>
<th>B₂</th>
<th>B₃</th>
<th>B₄</th>
<th>B₅</th>
<th>B₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁</td>
<td>1</td>
<td>1/6</td>
<td>1/2</td>
<td>1/3</td>
<td>2</td>
<td>1/4</td>
<td></td>
</tr>
<tr>
<td>B₂</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>B₃</td>
<td>2</td>
<td>1/3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1/4</td>
<td></td>
</tr>
<tr>
<td>B₄</td>
<td>3</td>
<td>1/4</td>
<td>1/2</td>
<td>1</td>
<td>4</td>
<td>1/3</td>
<td></td>
</tr>
<tr>
<td>B₅</td>
<td>1/2</td>
<td>1/7</td>
<td>1/3</td>
<td>1/4</td>
<td>1</td>
<td>1/7</td>
<td></td>
</tr>
<tr>
<td>B₆</td>
<td>4</td>
<td>1/2</td>
<td>4</td>
<td>3</td>
<td>7</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1. Evaluation index system on aircraft financial leasing risk for airlines
Table 3. B-C judgment matrix

<table>
<thead>
<tr>
<th>B1</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>B2</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>B3</th>
<th>C7</th>
<th>C8</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>1/4</td>
<td>1/5</td>
<td>C4</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>C7</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>C2</td>
<td>4</td>
<td>1</td>
<td>1/3</td>
<td>C3</td>
<td>1/5</td>
<td>1</td>
<td>1/2</td>
<td>C9</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>C3</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>C6</td>
<td>1/3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B4</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
<th>B5</th>
<th>C12</th>
<th>C13</th>
<th>B6</th>
<th>C14</th>
<th>C15</th>
<th>C16</th>
</tr>
</thead>
<tbody>
<tr>
<td>C9</td>
<td>1</td>
<td>6</td>
<td>7</td>
<td>C12</td>
<td>1/2</td>
<td>2</td>
<td>C14</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>C10</td>
<td>1/6</td>
<td>1</td>
<td>3</td>
<td>C13</td>
<td>2</td>
<td>1</td>
<td>C15</td>
<td>1/3</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>C11</td>
<td>1/7</td>
<td>1/3</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>C16</td>
<td>1/2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

After normalize the original matrix, we can get weight of all levels.

Table 4. Weight of the intermediate level

<table>
<thead>
<tr>
<th>indicators</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight</td>
<td>0.06</td>
<td>0.38</td>
<td>0.12</td>
<td>0.11</td>
<td>0.04</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 5. Weight of indicator level

<table>
<thead>
<tr>
<th>indicators</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight</td>
<td>0.1</td>
<td>0.28</td>
<td>0.62</td>
<td>0.65</td>
<td>0.12</td>
<td>0.23</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>indicators</td>
<td>C9</td>
<td>C10</td>
<td>C11</td>
<td>C12</td>
<td>C13</td>
<td>C14</td>
<td>C15</td>
<td>C16</td>
</tr>
<tr>
<td>weight</td>
<td>0.74</td>
<td>0.18</td>
<td>0.08</td>
<td>0.33</td>
<td>0.67</td>
<td>0.54</td>
<td>0.16</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Step 3. Consistency check

Although the judges did not require a consistency when the judgment matrix was constructed, any big deviate from the consistency was unaccepted. Therefore we needed a consistency check by mean of calculating CR. When CR < 0.10, it means the overall level order is in consistency. Or it needs to adjust indicators of the same level until the requirement of the overall order of levels met.

Take the indicator B1-C check as an example. According to the formulas (1), (2), (3), we can get $\lambda_{max}$, CI, CR one by one.

$$\lambda_{max} = \frac{1}{n} \sum_{j=1}^{n} b_j \omega_j$$  \hspace{1cm} (1)
On formula (1), \( \lambda_{\text{max}} \) was the largest eigenvalue required to be solved. To solve this value, we need to sum up all the product of the value in each row and its weight. The average of those values is \( \lambda_{\text{max}} \).

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

(2)

\[
CR = \frac{CI}{RI}
\]

(3)

On formula (2), \( n \) is the degree of this indicator. On formula (3), RI is the average random consistency indicator; RI value is different because of the different degree of the indicator system. In this paper, RI value is 0.58 when the degree is 3. Through the calculation, \( \lambda_{\text{max}} \) value of indicator B1-C is 3.09, CI value is 0.045, CR value is 0.078 < 0.10, so the judgment matrix has the satisfactory consistency. Similarly, \( \lambda_{\text{max}} \) value of indicator B2-C is 3.004, CI value is 0.002, CR value is 0.004 < 0.10, consistency check passed, \( \lambda_{\text{max}} \) value of indicator B4-C is 3.1, CI value is 0.05, CR value is 0.086 < 0.10, consistency check passed, \( \lambda_{\text{max}} \) value of indicator B6-C is 3.009, CI value is 0.0045, CR value is 0.008 < 0.10, consistency check passed. There is no consistency issue about indicator B3-C and B5-C because it has only two evaluation indicator.

**Step 4. To establish the score matrix of evaluation indicator sample**

In evaluation index system, \( X_{ij} \) is needed to transform qualitative indicator into quantitative indicator. In accordance with personal habit and convenience of calculation, indicator quantification can be achieved by establishment the ranking standard of evaluation index. The level of finance lease risk can be dividing into four grades, which are low risk, general risk, higher risk, and highest risk with assigned value 1, 2, 3, 4. If its ranking is between low risk and general risk, the evaluation should be 1.5. If its ranking is between general risk and higher risk, the evaluation should be 2.5. If its ranking is between higher risk and highest risk, the evaluation should be 3.5. Experts mark all indicators based on the ranking standard, and obtaining appraisal vector of indicator \( j \) of factor \( I \) is \( (d_{i1j}, d_{i2j}, \ldots, d_{imj}) \). So the comprehension evaluation vector of indicator \( j \) of factor \( I \) could be marked as \( d_{ij} = \frac{\sum_{j=1}^{m} d_{ij}}{m} \). \( l \) is the number of \( d_{ijk} \).

**Step 5. Confirm the evaluation gray classification and calculate the gray evaluation coefficient**

Evaluation gray is to confirm the ranking of the gray, gray number of gray classification and whitening weight function of gray number. The project risk divides into four grades corresponding with different whitening weight function and threshold.

Gray classification Of the first kind is highest risk ( \( e = 1 \) ), its gray number is \( \oplus 1 \in \{0, 4, + \infty \} \) and whitening weight function is \( f_1 \); Gray classification of the second kind is higher risk ( \( e = 2 \) ), its gray number is \( \oplus 2 \in \{0, 3, 6\} \) and whitening weight function is \( f_2 \); Gray classification of the third kind is general risk ( \( e = 3 \) ), its gray number is \( \oplus 3 \in \{0, 2, 4\} \) and whitening weight function is \( f_3 \); Gray classification of the fourth kind is low risk ( \( e = 4 \) ), its gray number is \( \oplus 4 \in \{0, 1, 2\} \) and whitening weight function is \( f_4 \); If \( V_{ije} \) is the gray evaluation coefficient of \( X_{ij} \) belongs to “e gray classification, then \( V_{ije} = \frac{\sum_{i=1}^{n} f(e) \cdot d_{ije}}{n} \). If
Vij is the gray evaluation coefficient of the project belongs to each gray classification, then

\[ f_1\left(d_{ijk}\right) = \begin{cases} 
\frac{d_{ijk}}{4} & d_{ijk} \in [0,4] \\
1 & d_{ijk} \in [4,\infty) \\
0 & d_{ijk} \notin [0,\infty] 
\end{cases} \quad f_2\left(d_{ijk}\right) = \begin{cases} 
\frac{d_{ijk}}{3} & d_{ijk} \in [0,3] \\
\frac{6-d_{ijk}}{3} & d_{ijk} \in [3,6] \\
0 & d_{ijk} \notin [0,6] 
\end{cases} \]

\[ f_3\left(d_{ijk}\right) = \begin{cases} 
\frac{d_{ijk}}{2} & d_{ijk} \in [0,2] \\
\frac{4-d_{ijk}}{2} & d_{ijk} \in [2,4] \\
0 & d_{ijk} \notin [0,4] 
\end{cases} \quad f_4\left(d_{ijk}\right) = \begin{cases} 
1 & d_{ijk} \in [0,1] \\
2-d_{ijk} & d_{ijk} \in [1,2] \\
0 & d_{ijk} \notin [0,2] 
\end{cases} \]

Step 6. Calculate the weight vector and weight matrix of gray assessment

Experts give the gray evaluation weight of indicator Cij, if it belongs to “e” gray classification, then it should be rije, and

\[ \text{rij} = \frac{V_{ij}}{\sum_{k=1}^{e} V_{ijk}} \]

There are four grades of gray classification with e equals 1,2,3,4. So the gray evaluation weight matrix of Xij for each gray classification can be expressed \( r_{ij} = (r_{i1}, r_{i2}, r_{i3}, r_{i4}) \), and the gray evaluation weight matrix of Xij for all gray classification is \( R_i \).

\[ R_i = \begin{bmatrix} 
\begin{bmatrix} r_{i1} \\
r_{i2} \\
r_{i3} \\
\vdots \\
r_{ij} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \\
\begin{bmatrix} r_{i1} \quad r_{i2} \quad r_{i3} \quad r_{i4} \\
r_{j1} \quad r_{j2} \quad r_{j3} \quad r_{j4} \end{bmatrix} \end{bmatrix} \]

Step 7. Comprehension assessment

The comprehension evaluation result of indicator Xij is Ai, and \( A_i = W_i \times R_i \), \( R = (A_1, A_2, A_3, \ldots A_m)^T \). Then the comprehension evaluation result of index Xi is A, and \( A = W \times R \).

Finally, the experts will assign value to each gray scale according to matter of fact. In this model, it has four grades of gray classification that is highest risk, higher risk, general risk and low risk, correspond to the value vector of gray scale is B and B equals \((4, 3, 2, 1)\). In view of the above, the comprehension evaluation of project risk level can be calculated, that is \( c = c = A \times B^T \). The project risk could be assessed by the value of C.

3. Application of the model

Airlines need to assess the project risk if it wants to introduce the airplane by finance lease. Six experts have been invited to assess the risk level of the project and give the evaluation. Experts should be use the evaluation grade standard to make the evaluation. We can get the weight matrix and weight vector as follows:
According to the basic process of gray comprehension evaluation, weight matrix can be calculated as follows (see table 6).

\[ D^T = \begin{bmatrix}
4 & 3.5 & 2 & 1.5 & 3.5 & 4 & 4 & 1 & 1 & 4 & 4 & 4 & 1.5 & 1.5 & 4 & 3 \\
4 & 3 & 2 & 1 & 4 & 4 & 3.5 & 2 & 1.5 & 3.5 & 3 & 4 & 1 & 1.5 & 3.5 & 2.5 \\
3.5 & 3.5 & 2.5 & 1.5 & 3 & 3.5 & 4 & 1.5 & 1 & 4 & 4 & 4 & 2 & 1 & 3.5 & 3 \\
3.5 & 3 & 1.5 & 2 & 3.5 & 3 & 3 & 2 & 1 & 3 & 3 & 3.5 & 1.5 & 1 & 4 & 4 \\
4 & 4 & 2 & 1.5 & 3.5 & 4 & 3 & 1 & 1.5 & 3.5 & 3.5 & 3 & 1 & 1.5 & 3 & 4 \\
4 & 3.5 & 2 & 2 & 3.5 & 3.5 & 1.5 & 1 & 4 & 4 & 4 & 2 & 2 & 4 & 3
\end{bmatrix} \]

According to the basic process of gray comprehension evaluation, weight matrix can be calculated as follows (see table 6).

**Table 6. Weight matrix and weight vector**

<table>
<thead>
<tr>
<th>B1</th>
<th>r_{11}</th>
<th>0.54</th>
<th>0.41</th>
<th>0.05</th>
<th>0</th>
<th>B4</th>
<th>r_{41}</th>
<th>0.16</th>
<th>0.21</th>
<th>0.32</th>
<th>0.32</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_{12}</td>
<td>0.43</td>
<td>0.43</td>
<td>0.15</td>
<td>0</td>
<td>r_{42}</td>
<td>0.49</td>
<td>0.42</td>
<td>0.09</td>
<td>0</td>
<td></td>
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</tr>
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<td>r_{13}</td>
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<td>0.32</td>
<td>0.44</td>
<td>0</td>
<td>r_{43}</td>
<td>0.47</td>
<td>0.42</td>
<td>0.11</td>
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<th>0.37</th>
<th>0.2</th>
<th>B5</th>
<th>r_{51}</th>
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<th>0.42</th>
<th>0.11</th>
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<td>0.43</td>
<td>0.15</td>
<td>0</td>
<td>r_{52}</td>
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<tr>
<td>r_{23}</td>
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<td>0.42</td>
<td>0.09</td>
<td>0</td>
<td>r_{53}</td>
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<th>0.43</th>
<th>0.13</th>
<th>0</th>
<th>B6</th>
<th>r_{61}</th>
<th>0.49</th>
<th>0.42</th>
<th>0.09</th>
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<td>0.24</td>
<td>0.35</td>
<td>0.24</td>
<td>r_{62}</td>
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<td>0.42</td>
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<td>r_{33}</td>
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<td>0.18</td>
<td>0</td>
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</tbody>
</table>

According to \( A_i = W_i \times R_i \), we can calculate the vector \( A \) for comprehension evaluation as follows:

\[
A = W \times R = \begin{bmatrix} 0.06 & 0.38 & 0.12 & 0.11 & 0.04 & 0.28 \end{bmatrix} \begin{bmatrix} 0.32 & 0.36 & 0.32 & 0 \\ 0.29 & 0.31 & 0.28 & 0.13 \\ 0.23 & 0.27 & 0.31 & 0.19 \\ 0.24 & 0.26 & 0.26 & 0.23 \\ 0.27 & 0.3 & 0.27 & 0.16 \\ 0.28 & 0.32 & 0.25 & 0.15 \end{bmatrix}^T = \begin{bmatrix} 0.27 & 0.3 & 0.27 & 0.14 \end{bmatrix}
\]

The risk of finance lease project has divided into four grade of gray classification, and the vector of gray classification is \((4, 3, 2, 1)\). We can get the comprehension evaluation of risk level, \( H = A \times (4, 3, 2, 1)^T = (0.27, 0.3, 0.27, 0.14) \times (4, 3, 2, 1)^T = 2.68 \). As a result, the finance lease project is tending to higher risk.

Further analysis on weight matrix shows that higher risk of this project is mainly due to economic environment followed by financial and residual risks. The decision maker could make the right measures of higher risk indicator to lower the overall risk level.

### 4. Conclusions and future work

This article establishes a multiple-level gray evaluation module by analyzing the risk factors of the aircraft financial leasing, does a quantitative research on the evaluation index by using analytic hierarchy process, evaluates compressively the risk level of aircraft financial leasing and get validated by an example in the end. The method is objective, accurate and practicable.

Also, there is a lot of room for improvement, including further optimizing method and features.
5. Acknowledgment

The article was supported by the social science major program of Tianjin Educational Commission (Grant No. 2011ZD033), Tianjin education science project of twelve five-years (Grant No. HE3001), and the social science planning program of Tianjin (Grant No. TJYT10-11).

6. References


