Back Projection Algorithm for Impulse Missile-Borne Spotlight SAR Imaging

Chen Si, Zhao Huichang, Zhang Shuning, Chen Yong

Abstract

In order to extract feature information of target, and give full play to the warhead’s strike ability, a back projection (BP) algorithm for impulse missile-borne spotlight synthetic aperture radar (SAR) imaging is proposed in this paper. It firstly establishes the impulse echo signal model for the missile-borne spotlight SAR platform at dive attack stage, and then combines with the precise ranging result of imaging region by the seeker, all echoes are back projected to imaging region at different data recording point. Finally all the back projection results are added coherently to derive the SAR image. The simulation results indicate that this method which needs no approximation can achieve a good target SAR image, and the algorithm is simple and easy to be implemented in engineering.

Keywords: Impulse, Missile-Borne, Spotlight SAR, Back Projection Algorithm

1. Introduction

Missile-borne SAR guidance technology is a high-tech combination of SAR and precision guidance technology. Due to capacity of all-weather and all time, high resolution and strong anti-stealth ability, it attracts more and more attention in the filed of modern guidance technology [1-7]. The movement of missile is very complicated in terminal guidance, the nonlinear range walk and range curvature of echoes are very large, so the traditional imaging algorithm is not valid for missile-borne SAR and a simpler and more effective one is needed. A polar format algorithm for the missile-borne spotlight SAR platform was presented in reference [8], but the transmitted signal is linear frequency modulated pulse there, not impulse, and the algorithm is too complicated which needs range compression and motion compensation. Reference [9] introduced an algorithm of impulse SAR imaging formation, but whether the algorithm is suitable for missile-borne SAR guidance system was not discussed there.

Therefore, in this paper, a BP algorithm for impulse missile-borne spotlight SAR is proposed. It utilizes the target range information measured by seeker to project the echoes back to imaging region in order to coherent-superposition, draws a 3-D diagram and 2-D projective diagram, that is the SAR diagram of the imaging region. This method which needs no approximation can achieve a good target SAR image, and the algorithm is simple and easy to be implemented in engineering.

2. Echo model of impulse missile-borne spotlight SAR target

2.1. Imaging scene of impulse missile-borne spotlight SAR

Missile takes high resolution strip map images of imaging region in early terminal guidance, the imaging mode changes from strip map mode to spotlight mode once the target is locked down and tracked down [10-11], the imaging scene is shown in figure 1.

Because of high speed of missile and short synthetic aperture time in the terminal guidance, the missile can be considered to dive with constant acceleration along the trajectory ABC. \( t_r \) denotes the range fast-time, \( t_a \) denotes the azimuth slow-time, the center of imaging region \((X_c, Y_c, 0)\) is defined as coordinate origin \( O \). Assume that the missile is located at \((x_0, 0, z_0)\) when \( t_a = 0 \), and it moves with a constant acceleration. The initial velocity along the \( x \) axis, \( y \) axis and \( z \) axis are \( v_x \), \( 0 \) and \( v_z \), respectively.
respectively, the constant acceleration are \( a_x \), 0 and \( a_z \), respectively. Assume that there is a point target located at \((X_T, Y_T, Z_T)\) in the imaging region. According to the geometry in figure 1, the transient distance between missile and target at any time during the synthetic aperture time can be written as

\[
R(t_a) = \sqrt{\left( x_0 - v_s t_a - \frac{1}{2} a_x t_a^2 - X_T \right)^2 + Y_T^2 + \left( z_0 - v_z t_a - \frac{1}{2} a_z t_a^2 - Z_T \right)^2}. \tag{1}
\]

Figure 1. Imaging scene diagram of missile-borne spotlight SAR

2.2. Echo signal model

Considering low centre frequency of impulse is not conductive to signal radiation and differential effect of the UWB antenna, the fourth derivative Gaussian pulse is chosen as the transmitted signal in this paper, it can be written as

\[
x(t) = \left( \frac{3}{\sqrt{2\pi}\sigma^3} - \frac{6t^2}{\sqrt{2\pi}\sigma^5} + \frac{t^4}{\sqrt{4\pi}\sigma^7} \right) e^{-\frac{t^2}{2\sigma^2}}, \tag{2}
\]

where \( \sigma \) denotes the pulse width factor.

The waveform of a single fourth derivative Gaussian pulse is shown in Figure 2.
Assume that there are $M$ point targets in the imaging region, the 2-D echo can be written as

$$s_r(t_r,t_a) = \sum_{i=1}^{M} s_i(t_r,t_a) + n(t_r,t_a) = \sum_{i=1}^{M} A_i \cdot s \left( t_r - \frac{2R_i(t_a)}{c} \right) + n(t_r,t_a),$$  \hspace{1cm} (3)

where $s_i(t_r,t_a)$ denotes the 2-D echo reflected from the $i$-th point target, $A_i$ is amplitude of the $i$-th 2-D echo, $R_i(t_a)$ is the transient distance between the $i$-th point target and the missile at time $t_a$, $c$ denotes the speed of light, $n(t_r,t_a)$ represents the 2-D Gaussian white noise.

3. Analysis of BP algorithm for impulse missile-borne spotlight SAR

3.1. Principle of BP imaging algorithm

BP algorithm is initially applied to computerized tomography (CT) technology. CT imaging has something in common with SAR imaging, so it also can be used in SAR imaging technology [12-13].

The principle diagram of BP imaging algorithm is shown in figure 3. For convenience, assume the missile moves in a straight line $AB$, the 2-D echo signal received by seeker is shown as heavy line $COD$, where the echo corresponds to point $C$ when the missile is located at point $A$, the echo corresponds to point $D$ when the missile is located at point $B$, by that analogy, the echo of each point on the synthetic aperture $AB$ corresponds to each point on the heavy line $COD$. $AO = AC$ and $BO = BD$ are easy to obtain by the principle of 2-D echo recording. According to precise ranging result of imaging region by the seeker, all echoes are back projected to imaging region at different data recording point, that are arcs which intersect at point $O$ shown in the figure. Finally all the back projection results are added coherently to derive the SAR image.
3.2. Steps of BP imaging algorithm for impulse missile-borne spotlight SAR

From the principle above, the algorithm includes following steps.

Step 1: The grid matrix of the imaging region $I(i, j)$ can be achieved by $i \times j$ meshing the imaging region. Then calculate the transient distance $R(i, j;n)$ between all the imaging grid cells and missile at $n$-th data recording point, where $n$ have a value from 1 to $N_a$ and transform $R(i, j;n)$ to range sampling number matrix $R_N(i, j;n)$ by $R_N(i, j;n) = R(i, j)/\Delta R$, where $\Delta R$ is the range sampling interval.

Step 2: Search the $n$-th echo signal $s_r(n, :)$ to achieve the echo delay $\tau$, and transform it to range sampling number $N_r$.

Step 3: Search the matrix $R_N(i, j;n)$ to back project the echo $s_r(n, :)$ to the imaging cell which $R_N(i, j;n)$ equals to $N$, the back projection result can be written as $I(i, j;n)$.

Step 4: Do the same with all the echo recording points from 1 to $N_a$.

Step 5: Sum all the back projection results $I(i, j;n)$ coherently, namely $G(i, j) = \sum_{n=1}^{N_a} I(i, j;n)$, where $G(i, j)$ is the BP imaging result. Then draw 3-D and 2-D diagram according to $G(i, j)$, that is the SAR image.

4. Simulation Results and Discussions

In order to make a further explanation for the method mentioned above, a simulation is conducted for an airplane. The simulation parameters are shown as below. Assume the center of the scene is the coordinate origin, the scene area in azimuth dimension is from -150m to 150m, and in range dimension is from -300m to 300m. Sampling numbers in azimuth dimension $N_a$ is 512 and in range dimension $N_r$ is 2048. Assume the missile is located at $(x_0, y_0, z_0) = (-1040, 0, 430)$ initially, and it
moves with constant acceleration in azimuth dimension and height dimension, the initial velocity $v_x$ is $500 \text{m/s}$ and the uniform acceleration $a_x$ is $20 \text{m/s}^2$ in azimuth dimension, and in height dimension, the initial velocity $v_z$ is $-200 \text{m/s}$ and the uniform acceleration $a_z$ is $-15 \text{m/s}^2$. The imaging range of the missile is from $-1040 \text{m}$ to $-100 \text{m}$, which means that synthetic aperture length is $940 \text{m}$. The fourth derivation Gaussian pulse with a pulse duration $T_p$ of $1 \text{ns}$ is taken as transmitted signal. Assume there is an airplane within imaging region which consists of 15 main scattering centers, their position matrix can be written as

$$
(x_T, y_T, z_T) = \begin{bmatrix}
0, -90, 0 & 0, 30, 0 & -15, 90, 0 & 15, 90, 0 & 0, 0, 0 \\
0, -60, 0 & 0, 60, 0 & -30, 30, 0 & 30, 30, 0 & 15, 0, 0 \\
0, -30, 0 & 0, 75, 0 & -45, 60, 0 & 45, 60, 0 & -15, 0, 0
\end{bmatrix}.
$$

Because of complicated motion in the terminal guidance, the echo is not the standard hyperbolic curve as shown in figure 3, but a piece of nonstandard one, shown in figure 4. From the figure 4, it can obviously see that the nonlinear range walk and range curvature of the echo are very large, so the traditional imaging algorithm is not valid for missile-borne SAR.

![Echo of the imaging region: (a) 3-D figure, (b) 2-D figure](image)

Figure 4. Echo of the imaging region: (a) 3-D figure, (b) 2-D figure

Follow the steps introduced in section 3.2, once processed by the BP imaging algorithm, a 3-D figure of the imaging region can be obtained, as shown in figure 5. There are 15 peak-points higher than other peaks in the figure clearly, namely 15 main scattering centers of the airplane. Figure 6 is the range-azimuth constant height chart of figure 5, and also it is the SAR image of imaging region, figure 7 and figure 8 are the range and azimuth direction response of the center of the scene. In figure 5, the coordinates of 15 peak-points can be obtained easily, and they correspond to the airplane’s position matrix assumed before. Thus, the SAR image for impulse missile-borne spotlight SAR can be achieved easily and effectively by using this algorithm.

However, in practical application, because of distance measuring error and other interference, the noise is inevitable, so the anti-noise capacity of this algorithm should be discussed. 2-D white Gaussian noise is added to the echo shown in figure 4(a), the new echo is shown in figure 9, then do the processing as above, the 3-D and 2-D SAR image of the imaging region can be obtained, as shown in figure 10 and figure 11, figure 12 and figure 13 are the range and azimuth direction response of the center of the scene with noise.
Compared figure 5 with figure 10, figure 6 with figure 11, figure 7 with figure 12, and figure 8 with figure 13, it can be seen that the noise degrades the image quality a litter, but can not interfere the SAR imaging obviously. Because the distribution of noise is always chaotic, after processing by the BP imaging algorithm, the point clustering by coherent superposition can not be achieved. In addition, finer grid division and better ranging method is conductive to image quality, but the finer grid division is, the heavier computation is, so it needs to be considered fully.

Figure 5. 3-D figure of the imaging region

Figure 6. 2-D figure of the imaging region

Figure 7. Range direction response of the center of scene region

Figure 8. Azimuth direction response of the center of scene region

Figure 9. Echo of the imaging region with noise: (a) 3-D figure, (b) 2-D figure
5. Conclusions

This paper proposed a BP algorithm for impulse missile-borne spotlight SAR imaging. It firstly utilizes the seeker to ranging the imaging region which takes the fourth derivation Gaussian pulse as transmitted signal, then combines with the precise ranging results, all echoes are back projected to imaging region at different data recording point, finally all the back projection results are added coherently, a 3-D diagram which the peaks points correspond to imaging region’s main scattering centers can be drawn, namely the SAR image of imaging region. The simulation results indicate that this method which needs no approximation can achieve a good target SAR image, and the algorithm is simple and easy to be implemented in engineering.

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7. References


