Information Fusion for Radar/Infrared Compound Seeker based on Federated Filter

Yan-ke Xu, Xiao-geng Liang


Abstract

Active radar/infrared (IR) compound guidance technology has become a hot research content of compound guidance field. According to the characteristics and the engineering application of active radar/IR composite seeker system, in this paper, a distributed flow of information fusion for Radar/IR composite seeker was established. First the observation data of the two seekers were pretreated, including time and space registration and outlier elimination of the observation data. After that, the Federated Filter (FF) was used to setup an information fusion algorithm of radar/IR composite seeker. According to the different characteristics of radar and IR system, the Extended Kalman Filter (EKF) algorithm and the Pseudo-linear Kalman Filter (PLKF) algorithm were used to design radar and IR local filter respectively. The simulation results show that this information fusion algorithm provides significant improvement in the tracking precision of the radar/IR composite seeker system, and it has good real-time performance and stability.

Keywords: Information Fusion, Compound Guidance, Federated Filter (FF), Time and Space Registration, Pseudo-linear Kalman Filter (PLKF)

1. Introduction

Interest in compound guidance technology has increased sharply over the last decade [1]. Because of the complicated battlefield environment, the single-mode seeker has almost been unable to meet the complex optical confrontation on the battlefield. At present, it is widely recognized that in the future, radar and infrared combination is the best approach to achieve intelligent guidance, it can generally improve the combat ability of the missile in all-weather conditions, it also improves the tracking precision and the anti-electronic jamming capability of the missile[2].

As the key technology of compound guidance, the information fusion technology has got many scholars' attention in recent years [3][4][5]. At present, there are two common data fusion structures: centralized and distributed. Now most information algorithms for the dual-model seeker were using centralized structure [6][7][8]. However, one problem of the centralized fusion algorithm is that it requires large amount of calculation as well as it has poor ability in fault-tolerance, so this structure is difficult to apply to practical engineering. According to the characteristics and the engineering application of active radar/IR composite seeker system, in this paper, a distributed flow of information fusion for Radar/IR composite seeker was established based on Federated Filter algorithm, the key technologies in this flow were discussed.

This paper is organized as follows. In section 1, the data preprocessing algorithm and data fusion algorithm are discussed. In section 2, we focus on a distributed information fusion algorithm based on Federated Filter (FF), a Federal Kalman tracking Filter with two-level structure is constructed. According to the different characteristics of radar system and IR system, we use the Extended Kalman Filter (EKF) algorithm and the Pseudo-linear Kalman Filter (PLKF) algorithm to design radar and IR local filter respectively, and then, the main filter is responsible for data fusion and data reset to local filters. In section 3, a simulation is made to compare the centralized EKF filtering algorithm and the FF filtering algorithm in this paper. Section 4 concludes the main contribution of the paper.
2. Radar and infrared preprocessing

2.1. Elimination of Wild Values

Because the radar/IR system has measurement errors and the environment is changing, the observation sequences of radar/infrared system contain some error data, we call them wild values. They are a small part of data points that gravely deviate from the tendency most of the data show. These wild values are far away from the true values of the target, if we don’t eliminate these wild values, the accuracy of seeker data processing system will be decreased.

In [9], a method that using the statistical properties of prediction residual to eliminate wild values was mentioned. The specific steps of this method are as follows:

\( \mathbf{Z}^{(1)}, \mathbf{Z}^{(2)}, \mathbf{Z}^{(k+1)}, \ldots, \) are the observations of the two seekers, let \( \mathbf{\hat{X}}^{(k+1)} \) denote the estimation of the target states \( \mathbf{X}(k+1) \). The prediction residual was denoted by \( \mathbf{v}^{(k+1)} \), so \( \mathbf{v}^{(k+1)} \) is expressed as

\[
\mathbf{v}^{(k+1)} = \mathbf{Z}^{(k+1)} - \mathbf{H}^{(k+1)} \mathbf{\hat{X}}^{(k+1)}/k
\]  

(1)

\[
\mathbf{H}^{(k+1)} = \left. \frac{\partial \mathbf{h}^{(k)}}{\partial \mathbf{X}(k+1)} \right|_{\mathbf{X}(k+1)=\mathbf{\hat{X}}^{(k+1)/k}}
\]  

(2)

If the observation noise is a zero mean Gaussian white noise, \( \mathbf{v}^{(k+1)} \) will also be a zero mean Gaussian white noise, and its diagonal covariance matrix satisfies

\[
\mathbf{S}^{(k+1)} = \mathbf{H}^{(k+1)} \mathbf{P}^{(k+1)}/k \mathbf{H}^{(k+1)}/k^T + \mathbf{R}^{(k+1)}.
\]  

(3)

where \( \mathbf{P}^{(k+1)}/k \) denotes the covariance matrix of the prediction error, \( \mathbf{R}^{(k+1)} \) is the covariance matrix of the observation noise. Using the statistical properties of \( \mathbf{v}^{(k+1)} \), we can judge each component of \( \mathbf{Z}^{(k+1)} \), the discriminant is as follows:

\[
\left| \mathbf{v}_{ij}^{(k+1)} \right| \leq C \sqrt{[\mathbf{S}^{(k+1)}]}_{jj},
\]  

(4)

where \( [\mathbf{S}^{(k+1)}]_{jj} \) is the \( i \)th element in the main diagonal of \( \mathbf{S}^{(k+1)} \), \( \mathbf{v}_{ij}^{(k+1)} \) is the \( i \)th element of \( \mathbf{v}^{(k+1)} \), \( C \) is a constant, usually taken to be 3 or 4. If \( \mathbf{v}_{ij}^{(k+1)} \) can satisfy (4), we will determine that \( \mathbf{Z}_{ij}^{(k+1)} \) is a correct observed value. Otherwise, we will use the extrapolate data from previous forecast to replace this observed value.

2.2. Time registration

Because the measurement of the two sensors are independent of each other, whilst they have different sampling period, the time that each sensor transmit data to the fusion center is different, so we need to process the measured data from different sensors to be synchronous. Common algorithms for time registration include least square method, interpolation & extrapolation method and curve fitting method. The least square algorithm, because of its good real-time performance, was chosen as the time registration algorithm used in this paper [10].

Suppose that the sampling period of radar seeker is \( T_1 \) and the IR seeker’s is \( T_2 \). And we know \( T_1 / T_2 = n \), that means in the time interval between two successive measurement of radar
seeker, the measurement number IR seeker can make is \( n \). For example, suppose the time interval between the \((k-1)\)th measurement and the \(k\)th measurement of radar seeker is \([t_{k-1}, t_k]\), and in this time interval, the IR seeker has its azimuth measured value \(\varphi\). Because the time interval of \([t_{k-1}, t_k]\) is very small, the changing rate of the measured value can be considered as a constant. Let \(x = [\varphi\; \dot{\varphi}]\) be the state vector at the time point \(t_k\), where \(\varphi\) is the azimuth measured by the IR seeker, and \(\dot{\varphi}\) is its change rate. Suppose \(\varphi_n\) is just the measured value at the time point \(t_k\), so

\[
\varphi(i) = \varphi + (i-n)T \dot{\varphi} + v_i.
\]

where \(v_i\) is the observation noise in the \(i\)th measurement and its mean is zero. We can define (5) to be a vector form as

\[
\varphi_n = H_n x + V_n.
\]

where \(H_n = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ -n & 1-n & \ldots & 0 \end{bmatrix}^T\). According to least square method, we can obtain the least square estimation of azimuth as follows:

\[
\hat{x} = [\hat{\varphi} \; \hat{\dot{\varphi}}] = (H_n^T H_n)^{-1} H_n^T \varphi_n.
\]

If we suppose that \(E(VV^T) = R\), the variance matrix of the estimation error of \(\hat{x}\) is expressed as:

\[
P_\varphi = (H_n^T H_n)^{-1} H_n^T R H_n (H_n^T H_n)^{-1}.
\]

So the estimation error of the azimuth is given by: \(R_\varphi = P_\varphi^{-1}\). Similarly, the least square estimation of pitch and the variance matrix of the estimation error are as follows:

\[
\hat{y} = [\hat{\theta} \; \hat{\dot{\theta}}] = (H_n^T H_n)^{-1} H_n^T \theta_n,
\]

\[
P_\theta = (H_n^T H_n)^{-1} H_n^T R H_n (H_n^T H_n)^{-1},
\]

\[
R_\theta = P_\theta^{-1}.
\]

### 2.3. Space registration

As the radar seeker and the IR seeker have different measuring coordinate systems, before data fusion, we need to transform the measured value of the two seekers into the same coordinate system. As long as we know the relative position and the definition of the two seekers’ measuring coordinate systems, we can do this transformation.

For example, define the measuring coordinate system \(O_aX_aY_aZ_a\) of radar seeker as follows (the definition of IR seeker’s is similar): the origin of the measuring coordinate system is the rotation center of the radar antenna, the \(O_aX_a\) axis denotes the antenna axis (in IR system, it

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denotes the optical axis), we define that at the moment of the missile launch, the measuring coordinate system coincides with the body coordinate system \( OXYZ \). When the seeker starts working, the antenna axis will rotate following with the line of sight, \( O_aY_a \) and \( O_aZ_a \) can be obtained by coordinate rotation.

Define the angle between the plane \( O_aX_aY_a \) and \( OXY \) to be \( \alpha \), when we observe from the tail of the missile to the head of the missile, if the \( O_aX_a \) axis is on the left side of the plane \( OXY \), \( \alpha > 0 \). Otherwise, \( \alpha < 0 \).

Define the projection line of \( O_aX_a \) axis in the plane \( OXY \) is \( O_aX_a' \), the angle between \( O_aX_a' \) and \( OX \) is \( \beta \), when we observe along the negative direction of \( OZ \) axis, if we can rotate \( OX \) to \( O_aX_a' \) in the minimum angle and the rotation direction is counterclockwise, \( \beta > 0 \). Otherwise, \( \beta < 0 \).

According to these definitions, we can give the transformation equation between the measuring coordinate system of the radar seeker and the body coordinate system [11]:

\[
\begin{bmatrix}
X_a \\
Y_a \\
Z_a
\end{bmatrix} =
\begin{bmatrix}
\cos\beta & \cos\alpha & \sin\beta & -\cos\beta\sin\alpha \\
-\sin\beta & \cos\alpha & \cos\beta & \sin\beta\sin\alpha \\
\sin\alpha & 0 & \cos\alpha
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

(12)

Similarly, we can give the transformation equation between the measuring coordinate system of the IR seeker and the body coordinate system, so we can transform the measured value of the two seekers into the same body coordinate system. Figure 1 shows the relationship between the body coordinate system of the missile and the radar/IR measuring coordinate system.

![Figure 1. Relationship between the body coordinate system of the missile and the radar/IR measuring coordinate system](image)

3. Fusion algorithm based on federated filter

Figure 2 is the information fusion structure diagram of the radar/IR compound seeker based on Federated Filter, where Space & time registration module provides each local filter with information on space-time registration. The estimation value \( \hat{X}_i \) and its covariance matrix \( P_i \) of each local filter are sent to the main filter, then the main filter integrates them with the estimation value \( \hat{X}_m \) and its
covariance matrix $P_m$ of the main filter, then we can get the global optimal estimation value $\hat{X}_k$ and its covariance matrix $P_k$. In the mean time, $P_k$ are distributed at the appropriate proportion and fed back to each local filter, so we can reset the estimated value of each local filter [12][13][14]. The rule of information distribution is as follows:

$$\hat{X}_i = \beta_i^{-1} \hat{P}_i (i = 1, 2)$$  \hspace{1cm} (13)

where $\beta_i$ is the information distribution coefficient,

$$\sum_{i=1}^{2} \beta_i + \beta_m = 1.$$  \hspace{1cm} (14)

After the information distribution, the two local filters accomplish the time & measurement update and the new measurement values are sent to the main filter, the fusion algorithm is expressed as:

$$P_k^{-1} = \sum_{i=1}^{2} P_{i,k}^{-1} + P_{m,k}^{-1}.$$  \hspace{1cm} (15)

$$\hat{X}_k = \sum_{i=1}^{2} P_k P_{i,k}^{-1} \hat{X}_{i,k} + P_k P_{m,k}^{-1} \hat{X}_{m,k}.$$  \hspace{1cm} (16)

Figure 2. Information fusion chart of radar/IR compound seeker system

3.1. State equation of the system

In the target tracking problem, creating a realistic, objective and easy mathematical treatment of the motion target is the key technology which makes the tracking algorithm to be applied. In the simulation of this paper, we use current statistical model to describe the motion state of the target. In this model, the state vector of the target can be:

$$X = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} & \ddot{x} & \ddot{y} & \ddot{z} \end{bmatrix}^T$$

where $x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}$ are the position, velocity and the acceleration of the target respectively, in this model, the state equation of the target is:

$$X_k = \Phi_{k,k-1} X_{k-1} + u \ddot{X}_{k-1} + W_{k-1}$$  \hspace{1cm} (17)
where $W_{k-1}$ is a zero mean Gaussian white noise, and its covariance matrix is $Q_w$. The meaning of other parameters can be found in reference [9].

### 3.2. Design of radar local filter

The observation values of radar seeker include azimuth, pitch, the distance between missile and target, etc (the observation is under the measuring coordinate system of the radar seeker). Let $Z_1(k) = [\varphi_R \ \theta_R \ r]^T$ denote the observation vector of radar seeker, where $\varphi_R$ denotes the azimuth, $\theta_R$ denotes the pitch, and $r$ denotes the distance between missile and target. Using our notation defined above, we can get the observation equation of radar seeker:

$$Z_1(k) = h_1(X_k) + V_1(k).$$

where $V_1(k)$ is a zero mean Gaussian white noise and its covariance matrix is $R_1$.

$$R_1 = \begin{bmatrix}
\sigma^2_{\varphi_R} & 0 & 0 \\
0 & \sigma^2_{\theta_R} & 0 \\
0 & 0 & \sigma^2_r
\end{bmatrix},$$

$$h_1(X_k) = \begin{bmatrix}
- \arctan(z) \\
\arctan(x) \\
\sqrt{x^2 + y^2 + z^2}
\end{bmatrix}.$$

Equation (17) and (18) form the subsystem of radar seeker, according to this subsystem, we can use Extended Kalman Filter (EKF) to estimate the motion state of the target. The filtering steps are as follows:

1. Initialization: give the initial value $\hat{X}_0, P_0$.
2. Prediction: $\hat{X}_{k,k-1} = \Phi_{k,k-1} \hat{X}_{k-1} + u\bar{a}_k$.
3. Calculate the prediction error matrix: $P_{k,k-1} = \Phi_{k,k-1} P_{k-1} \Phi_{k,k-1}^T + Q_w$.
4. Calculate the gain matrix of the filtering: $K_k = P_{k,k-1} H_k^T [H_k P_{k,k-1} H_k^T + R_k]^{-1}$.
5. Calculate the error matrix of the filtering: $P_k = [I - K_k H_k] P_{k,k-1}$.
6. Update the status value of the target: $\hat{X}_k = \hat{X}_{k,k-1} + K_k [Z_k - h_1(\hat{X}_{k,k-1})]$.

where $H_k = \frac{\partial h_1}{\partial \hat{X}}|_{\hat{X} = \hat{X}_{k,k-1}}$.

So we can obtain the estimation of state value $\hat{X}_{1,k}$ and covariance matrix $P_{1,k}$ using the radar local filter.

### 3.3. Design of IR local filter

The observation values of IR seeker include azimuth, pitch (the observation is under the measuring coordinate system of the radar seeker). Let $Z_2(k) = [\varphi_{IR} \ \theta_{IR}]^T$ denote the observation vector of the IR seeker, where $\varphi_{IR}$ denotes the azimuth, $\theta_{IR}$ denotes the pitch. Using our notation defined above, we can get the observation equation of IR seeker:
\[ Z_{2,k} = h_2(X_{2,k}) + V_{2,k}. \] \tag{19}

where \( V_{2}(k) \) is a zero mean Gaussian white noise and its covariance matrix is \( R_{2,IR} \).

\[
R_{2,IR} = \begin{bmatrix}
\sigma_{\theta_m}^2 & 0 \\
0 & \sigma_{\phi_m}^2
\end{bmatrix},
\]

\[
h_2(X_{2,k}) = \begin{bmatrix}
-\arctan\left(\frac{x}{y}\right) \\
\arctan\left(\frac{y}{\sqrt{x^2 + z^2}}\right)
\end{bmatrix}.
\]

Since the IR seeker can only detect the angle in formation but can’t detect the distance between missile and target, we can’t make right estimation about the motion state of the target using (17) and (19) as we did in radar local filter. So we must transform the system equations. In this paper, the Pseudo-linear Kalman Filter (PLKF) method is used to design the angular only tracking subsystem of the IR seeker.

According to [15], we use pseudo-linearization method to transform (19), so we can get:

\[ Z_{IR,k} = H_k X_{k} + n_k. \] \tag{20}

where

\[
H_k = \begin{bmatrix}
\sin\phi & 0 & 0 & 0 & 0 \\
\cos\phi & 0 & 0 & 0 & 0 \\
\cos\phi\sin\theta & -\cos\theta & -\sin\phi\sin\theta & 0 & 0
\end{bmatrix},
\]

\( Z_{IR,k} = 0 \), \( n_k \) denotes the effective measurement error which is a zero mean Gaussian white noise and its covariance matrix is \( R_z \):

\[
R_z = \begin{bmatrix}
r_z^2\sigma_{\phi_m}^2 & 0 \\
0 & r_z^2\sigma_{\theta_m}^2
\end{bmatrix},
\]

\[ r_z = \sqrt{\hat{x}_k^2 + \hat{z}_k^2}, \quad r = \sqrt{\hat{x}_k^2 + \hat{y}_k^2 + \hat{z}_k^2}. \]

So (17) and (20) form the subsystem of IR seeker, according to this subsystem, we can use Kalman Filter to estimate the motion state of the target. Then we can obtain the estimation of state value \( \hat{X}_{2,k} \) and covariance matrix \( P_{2,k} \) using the IR local filter.

3.4. Design of main filter

Let the information distribution coefficient of the main filter \( \beta_m = 0 \), so we can obtain that \( P_{m}^{-1} = 0 \). That means the main filter will not do filtering but only integrate the information from each local filter. Thus, the estimation values of the motion state in the main filter are:

\[
P_{k}^{-1} = \sum_{i=1}^{2} P_{i,k}^{-1} \quad \hat{X}_{k} = \sum_{i=1}^{2} P_{i,k}^{-1} \hat{X}_{i,k}. \] \tag{21}
According to the rule of information distribution, the global optimal estimation value $\hat{X}_k$ and the covariance matrix $P_k$ are fed back to every local filter to reset their estimated value. That is:

$$\hat{X}_{i} = \hat{X}_k, \quad (22)$$

$$P_{i} = P_k^{-1} P_k, \quad (23)$$

$$Q_{i} = Q_k^{-1} Q_k. \quad (24)$$

### 3.5. Analysis of system computation

The computation of the classical EKF algorithm mainly concentrates on the matrix inversion. The centralized EKF filtering algorithm uses parallel filtering method; the increasing of its matrix dimension will cause the increasing of the system computation. The distributed information fusion algorithm in this paper does not expand the dimension of the system state vector, therefore, compared with the centralized information fusion algorithm, the algorithm in this paper can significantly reduce the system computation and have good real-time performance.

### 4. Simulations and analysis

Suppose the target maneuver model is S maneuver. In the ground coordinate system, the initial position of the target is $(15000, 8000, -8000)$ m, the initial velocity of the target is $(200, -50, 0)$ m/s and the initial acceleration of the target is $(0, 5, 0)$ m/s$^2$. Suppose the noise variance of the initial state of the target is $\sigma^2_x = \sigma^2_y = \sigma^2_z = 0.1 \times 10^{-4}$. Suppose the initial position of the missile is $(0, 0, 0)$ and its velocity is $340$ m/s, its initial movement direction is parallel to the line of sight. We use proportional navigation (PN) method to guidance the missile and the proportional coefficient is $N = 4$. The initial noise variance of radar seeker is:

$$R_i = \begin{bmatrix} 0.004 \text{rad}^2 & 0 & 0 \\ 0 & 0.004 \text{rad}^2 & 0 \\ 0 & 0 & 400 \text{m}^2 \end{bmatrix}$$

The initial noise variance of radar seeker is:

$$R_{s,IR} = \begin{bmatrix} 4 \times 10^{-4} \text{rad}^2 & 0 \\ 0 & 4 \times 10^{-4} \text{rad}^2 \end{bmatrix}$$

Suppose the sampling time of the two seekers are both $0.02$ s, by Monte Carlo simulation, the simulation results are as follows:
Figure 3. The moving trace of missile and target

Figure 4. The comparison of X axis position error by using single mode data processing system/dual-mode data fusion system

Figure 5. The comparison of Y axis position error by using single mode data processing system/dual-mode data fusion system
Figure 6. The comparison of Z axis position error by using single mode data processing system/dual-mode data fusion system

Figure 7. The comparison of X axis position error by using centralized EKF algorithm/FF algorithm

Figure 8. The comparison of Y axis position error by using centralized EKF algorithm/FF algorithm
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Figure 9. The comparison of Z axis position error by using centralized EKF algorithm/FF algorithm

Figure 3 shows the relative motion track of missile and target under the earth coordinate system, from the figure, we can see the data fusion algorithm in this paper can track target very well. Figure 4-6 are the comparisons of position error by using single mode data processing system and federal filtering data fusion system, obviously, the federal filtering data fusion system can provide significant improvement in the tracking precision. Also, because the IR seeker can’t detect any distance information and it is angular only tracking, with the increase of the simulation time, the estimation error of the IR seeker becomes larger, the filtering divergence problem appeared. But using the federal filtering data fusion, we can overcome this problem effectively and the stability of the system has been improved.

Figure 7-figure 9 are the comparisons of position error by using centralized EKF filtering algorithm and FF filtering algorithm. From the figure, we observed that the tracking precision of the FF fusion system is as good as the traditional centralized fusion system, so the FF filtering algorithm can meet the system requirements of tracking precision. Whilst, compared with the centralized EKF filtering algorithm, the FF filtering algorithm can significantly reduce the system computation and has good real-time performance.

5. Conclusions

Starting from the engineering application of dual-mode seeker, a distribution flow of information fusion for Radar/IR composite seeker was established. We use the federated filter (FF) to set up an information fusion algorithm of radar/IR composite seeker. The theoretical research and the simulation experiments both show that compared with the centralized fusion algorithm, the distributed FF fusion algorithm can reduce the system computation and has good real-time performance, therefore, the radar/IR guidance system is technically feasible by using the FF fusion algorithm.

In this paper, the federal filtering fusion algorithm is used to the radar/IR dual-mode seeker system. According to the different characteristic of radar and IR system, the Extended Kalman Filter(EKF) algorithm and the Pseudo-linear Kalman Filter(PLKF) algorithm are used to design radar and IR local filter respectively. The simulation results show that this information fusion algorithm provides significant improvement in the tracking precision of the radar/IR composite seeker system, also it can overcome the filtering divergence problem of the IR seeker and improve the stability of the system.
6. References